Robust MAV State Estimation Using an M-Estimator Augmented Sensor Fusion Graph

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ABSTRACT

INTRODUCTION

The use of Micro Air Vehicles or MAVs for commercial applications has grown drastically in the recent years. Due to their capabilities of omnidirectional flight, vertical take-off, and hovering, MAVs are versatile platforms that can operate efficiently within urban environments. Most commonly MAVs are tasked with navigating through human-centric and urban environments for consumer product delivery and aerial photography. However the proximity of operating in human-centric environments present challenges to the GPS-based navigation systems commonly used in autonomous outdoor navigation. Before widespread commercial adoption can occur, MAV navigation needs to be safe and reliable.

In this paper we address the three primary challenges for MAV navigation in GPS-challenged environments. Due to the cluttered and obstacle-rich environments of human-centric navigation, MAV localization needs to be frequent accurate. Often times, GPS errors can be up to tens of meters. In an environment where obstacle avoidance is key to the survival of the system, a large positional error can result in loss of the MAV. In addition, the GPS measurement rate is too slow for autonomous control of an MAV. In order to maintain position control, a navigational sensor with a high measurement rate such as an Inertial Measurement Unit (IMU) is needed. Navigating in urban environments also bring the challenge of urban canyons. Due to prolific structures in urban environments, satellite visibility from user receivers are often occluded. Since at least four satellites are required for a navigation solution, GPS alone is not sufficient for reliable and consistent positioning. Finally structures and other obstacles can also attribute to another source of GPS error, multipath and Non-Line Of Sight (NLOS) errors. These occur when GPS receivers receive reflected signals bounced off of the nearby environment. The inconsistent travel time misrepresents the true distance from the satellite to the receiver and as a result introduces an outlier type error in the GPS derived navigation solution.

In our approach, we present a graph based approach to sensor fusion of GPS and IMU measurements for accurate and frequent positioning. In prior work, Kalman type filters have been commonly used for GPS/IMU sensor fusion[1][2]. Daellert and Kaess introduced the smoothing approach as a viable alternative to extended Kalman filters for positioning a robot’s path through landmark measurements and a motion model [3]. Bryson et al. presented a smoothing approach to IMU, GPS, and monocular vision fusion for large-scale terrain reconstructions [4]. In this paper we present a smoothing approach for integrating measurements from low-cost GPS and IMU sensors with a simple UAV dynamics model. We then augment our sensor fusion graph with m-estimators for a robust navigation solution in the presence of GPS outliers.

First proposed by Huber [1] in 1964, m-estimation has been widely used for robust statistical estimation. Robust estimation schemes are not new to the field of GNSS navigation and have been used to handle GNSS sensor outliers. Gui et al uses m-estimators in order to establish a series of robust biased estimators [2]. Knight et al compares different GNSS outlier detection, rejection, and robust estimation techniques including M-estimators. Chang et Al [Huber’s M-Est] applies M-estimation to code and car-
rrier phase based relative position. Typically M-estimators are applied using Kalman-type filters. Perala and Pesonen both robustify the Kalman filter by implementing a re-weighted Kalman type filter for GNSS positioning. Our approach ties m-estimation together with the Smoothing-and-Mapping approach specifically for MAV navigation.

**APPROACH**

We define our MAV state as the position, velocity, and acceleration of the MAV in the Earth Centered Earth Fixed (ECEF) coordinate frame along with the IMU accelerometer and GPS clock biases. We then tightly couple the IMU and GPS code measurements in a sensor fusion graph.

We outline our approach in four major segments. First we formulate the navigation challenge as a belief network and model the trajectory of the MAV using a probabilistic graph. From the graph, we form a joint probability model representative of the trajectory state estimates of the MAV. Using Maximum a Posteriori (MAP) estimation, we find the most likely probabilistic trajectory of the MAV. To solve the mathematical complex problem, we use numerical methods to iteratively solve of our optimization parameters. Finally since the first few steps assume Gaussian measurements and processes and GPS outliers are commonly non-Gaussian, we use m-estimation to mitigate these sensor outlier and de-weight faulty GPS measurements on our navigation state solution.

**Probabilistic Graph**

Our MAV trajectory is modeled as a Bayesian network, specifically a directed acyclic graph. Each edge points in a corresponding direction and there are no loops formed by series of nodes. The graph denotes the conditional dependencies between the random variables in the MAV trajectory estimation problem. Each node represents a measurement or a state and each edge represents a directed dependency. We describe the set of all nodes as the trajectory of the MAV. Since the IMU is the sensor with the highest frequency, we use the IMU measurement rate as the foundation of our state nodes. We then incorporate GPS measurements at corresponding time steps where GPS measurements are available. These act as anchor nodes that aid in IMU bias estimation and correct for IMU sensor drift. Each node is connected to its predecessor through an edge representing the propagation of the MAV state from its prior step to its next using a dynamics model. We formulate a joint probability model from the GPS probabilistic graph. For a GPS system of $T$ time steps with $K$ satellites in view, we denote the joint probability of the

**MAV trajectory state estimation as:**

$$P(X, Z, A, dt, \beta) =$$

$$P(x_0) \prod_{i=1}^{T} P(x_i|x_{i-1}) \prod_{i=1}^{K} P(p_i^k|x_i, dt) \prod_{i=1}^{T} P(\dot{x}_i|\alpha_i, \beta) \tag{1}$$

where $P(X, Z, A, dt, \beta)$ is the joint probability of the trajectory for the set of all MAV states $X$, set of all pseudorange measurements $Z$, and set of all accelerometer measurements $A$. $P(x_0)$ is a prior on the initialization state, representing certainty from the MAV pre-takeoff state. $P(x_i|x_{i-1})$ is the probability of the $i^{th}$ MAV state propagated from the previous state using the MAV motion model. $P(p_i^k|x_i, dt)$ denotes the probability of observing a specific GPS measurement or pseudorange $p_i^k$ at satellite $k$ from the current MAV position $x_i$ with the current receiver clock bias estimate $dt_i$. $P(\dot{x}_i|\alpha_i, \beta)$ denotes the IMU measurement model where the acceleration estimate $\dot{x}_i$ is can be defined as a function of the measured acceleration $\alpha_i$ and the accelerometer bias $\beta$.

Assuming Gaussian processes and measurements, we generate probability density functions (pdfs) with normal distribution:

$$P(X) = \exp^{-\frac{1}{2}}||g(x)||_2^2 \tag{2}$$

where $g(x)$ is the corresponding process or measurement model and $\Sigma$ is the error covariance matrix.

We define our dynamics propagation process model $f(x)$ as composed of two parts, $f_{position}(x)$ and $f_{velocity}(x)$:

$$f_{position}(x) = x_{i-1} + \dot{x}_{i-1} \delta t + \frac{1}{2} \ddot{x}_{i-1} \delta t \tag{3}$$

$$f_{velocity}(x) = \ddot{x}_{i-1} + \dot{x}_{i-1} \delta t \tag{4}$$

The dynamics propagation model is associated with the current states as:

$$x_i = f(x_{i-1}) + w_i \tag{5}$$
for a normally distributed zero-mean process noise $w_t$ with covariance $\Sigma_D$. We model GPS measurements as:

$$r(\hat{x}_i, dt_i) = \sqrt{(x_i^k - x_i)^2 + (\dot{x}_i - y_i)^2 + (z_i^k - z_i)^2 + c dt_i}$$

(6)

where $c$ is the speed of light and $dt_i$ is receiver clock bias at time $i$ and the pseudorange measurement $\rho_i^k$ at time $i$ to satellite $k$ as:

$$\rho_i^k = r(x_i, dt_i) + \epsilon_i^k$$

(7)

The pseudorange error is represented as $\epsilon_i^k$ with covariance $\Sigma_p$. IMU measurements are modeled by:

$$\dot{x}_i = \alpha_i + \beta_i + \mu_i$$

(8)

where the accelerometer errors are modeled as $\mu_i$ with covariance $\Sigma_\alpha$. The joint probability model can be represented as a product of pdfs.

$$P(x_i|x_{i-1}) \propto \exp\left(-\frac{1}{2}||f(x_{i-1}) - x_i||^2_{\Sigma_D}\right)$$

(9)

$$P(\rho_i^k|x_i, dt_i) \propto \exp\left(-\frac{1}{2}||\rho_i^k - r(x_i, dt_i)||^2_{\Sigma_p}\right)$$

(10)

$$P(\dot{x}_i|\alpha_i, \beta_i) \propto \exp\left(-\frac{1}{2}||\dot{x}_i - (\alpha_i + \beta_i)||^2_{\Sigma_x}\right)$$

(11)

We can then define our joint probability model as a product of pdfs.

**Maximum a Posteriori (MAP) Estimation**

By formulating the MAV trajectory estimation problem as a joint probability where we seek to find the most probable trajectory, we can approach this as an optimization problem of pdfs. We optimize for the most probable trajectory parameterized by $\Theta \equiv (X, Z, A, dt, \beta)$:

$$\hat{\Theta} = \arg \max_\Theta P(X, Z, A, dt, \beta) = \arg \min_\Theta \log(P(X, Z, A, dt, \beta))$$

(12)

Since the logarithm is a monotonically increasing function, we can take the negative log-likelihood function and transform our maximization problem into a minimization of a series of least squares.

$$\hat{\Theta} = \arg \min_\Theta \left\{ \sum_{i=1}^T ||f(x_{i-1}) - x_i||^2_{\Sigma_D} + \sum_{i=1}^T \sum_{k=1}^K ||\rho_i^k - r(x_i, dt_i)||^2_{\Sigma_p} + \sum_{i=1}^T ||\dot{x}_i - (\alpha_i + \beta_i)||^2_{\Sigma_x} \right\}$$

(13)

**Iterative Least Squares (ILS)**

Due to the mathematical complexity of the minimization problem, we use an iterative method to solve for the state estimation parameters. Consider the linear regression problem:

$$y = Hx + v$$

(14)

where $y$ is the vector of observations, $H$ is the design matrix or the Jacobian, $v$ is the observation error with variance $\sigma$, and $x$ is the vector of parameters that minimize the optimization problem. We can formulate the design matrix as a stacked block matrix of the Jacobians of each process and measurement used in the sensor fusion graph. Although vast, the design matrix $H$ as shown in Figure 2 remains sparse and thus computationally efficient to solve. We then define our quadratic cost function as $c(x) = y - Hx$.

The cost vector $c(x)$ is a stacked vector of the residuals from each process and measurement. We achieve the minimum with the parameter estimate:

$$\hat{x} = (H^T\Sigma^{-1}H)^{-1}H^T\Sigma^{-1}y$$

(15)

In order to numerically solve for a solution, we use the iterative process until numerical convergence:

$$x_{{iter+1}} = x_{{iter}} + (H^T\Sigma^{-1}H)^{-1}H^T\Sigma^{-1}c$$

(16)

**M-Estimation**

In ordinary linear regression, processes and measurements are assumed to be Gaussian. The associated the probability distribution function can be derived as:

$$P(x) = e^{-||x||^2}$$

Through the process of MAP estimation, it is shown that the quadratic cost function is inherently built into least squares solutions. Thus we call the quadratic cost function a statistical cost function due to its characterization of errors dependent on a Gaussian approximation. However NLOS, Multihop, and other sensors are non-Gaussian. Outliers dramatically increase the objective constraint of a system, skewing the resultant solution. In order to mitigate the effects of outliers, we bound their influences through the introduction of robust cost functions or m-estimators.

Instead we consider a heuristic cost function where the function costs are characterized by function heuristics and noise-immuneness properties rather than adherence to a specific noise distribution model [9]. Specifically we choose the Huber cost function, a hybrid of the L1 and
quadratic cost functions:

\[ C_{\text{Huber}}(\delta) = \begin{cases} \frac{\delta^2}{2} & |\delta| \leq b \\ |\delta| - \frac{\delta^2}{2} & |\delta| > b \end{cases} \]  

where \( \delta \) is the cost parameter and \( b \) is the threshold parameter. The L1 cost function is robust in that it is resistant to outliers in data, however its solution is unstable and may not be unique. On the other hand, the quadratic cost function has a stable unique solution and is differentiable at the origin. Its downside however is that it is not very robust to outliers in data. By using the Huber cost function, we can draw upon the stability and uniqueness of the quadratic cost function and the robustness of L1. We apply m-estimation to the traditional least squares problem using a weighted cost function. The minimization of the squared vector norm \(|\delta|^2\) is inherently built into the implementation of iterative least squares. We replace each measurement residual \( \delta \) with a weighted residual \( \delta' = w\delta \) such that

\[ ||\delta||^2 = w^2||\delta'||^2 = C_{\text{Huber}}(\delta) \]

\[ w = \frac{C_{\text{Huber}}(\delta)}{\delta} \]

The weighting function \( w \) can be specified as an attenuation function that attenuates the cost of the outliers. In comparison, for the standard quadratic error cost function, the attenuation factor is one, meaning no attenuation occurs and its original value is used.

**EXPERIMENTAL RESULTS**

Experiments were conducted on a MAV in a GPS challenged environment to evaluate the performance of our m-estimator augmented sensor fusion graph on handling GPS outliers.

Flight data was collected from a Ascending Technologies (AscTec) Firefly Hexacopter. The AscTec Firefly was equipped with an AscTec Mastermind with an Intel® Core™ i7-3612QE Quad core running at 2.1 GHz for data collection. An external GPS receiver, the ublox LEA-6T GPS receiver, with a Maxtena Passive GPS antenna was connected to the Mastermind computer and installed on the top of the MAV. The Mastermind interfaces with the Low Level Processor (LLP) of the on-board microprocessor of the Firefly for IMU accelerometer measurements. Flight data was logged on the AscTec Mastermind for post-flight analysis. The flights were conducted on the Bardeen Engineering Quad on the University of Illinois campus. The MAV was manually piloted at an altitude between three to five feet off the ground and remained tethered to a ground operator throughout the flight. Fig 6 shows the desired true trajectory where we sought to pilot the MAV, between a meter to two above the concrete path. However due to wind and other piloting conditions, the true trajectory is close to but not perfectly centered on the desired trajectory. The testing environment starts off in an open skied area with up to eight satellites in view, PRNs 2, 5, 13, 15, 20, 21, 29. The MAV was then navigated to a GPS-challenged area, near an alleyway between two tall buildings, the Grainger Engineering Library and the Mechanical Engineering Laboratory and back to the original take-off location. Fig 5 presents a relative scale of the alleyway along with the height of the buildings.

**Navigation in Good GPS Conditions**

We first verify that our algorithm provides consistent positioning results under good GPS conditions. We chose to highlight a section of our flight tests where 7-8 satellites were in constant view under no multipath or NLOS effects.
Figure 7 shows a comparison of the results from a GPS only trajectory vs. one that has been filtered using our GPS/IMU sensor fusion graph. The positional results have been plotted on a Google Map to show relative scale and comparison to the actual flight path. It can be seen from this segment that under optimal GPS conditions, the navigational solutions obtained from the GPS only vs. the GPS/IMU sensor fusion graphs are very similar. Figures 8 and 9 illustrate the distance of the MAV from its originating takeoff location. From these figures, we can see that the GPS/IMU sensor graph is able to provide a smoother trajectory estimate than that of GPS alone. The addition of the IMU constrains the GPS measurements and reduces the variance of the navigation solution upon the true trajectory.
Navigation in a GPS-Challenged Environment

Next we examine data collected during the GPS-challenged segment of our flight. We chose to focus on a segment where the MAV is flying near the front of the alleyway. As a result, PRN 2 has been completely occluded and PRN 15 is subject to multipath. Figure ?? shows a comparison of the GPS only, GPS/IMU graph, and robust GPS/IMU graph navigational solutions. Again, the positional results have been plotted on a Google Map to show relative scale and comparison to the actual flight path. Figures 13 and 13 show that GPS outliers can cause an error up to 20m in the navigation solution. With the sudden spike in navigation solution, if the system was flying autonomously, relying on GPS measurements alone, the MAV would be on a collision course with the building. After incorporating the GPS/IMU sensor fusion graph, it is shown that the GPS outlier errors could be reduced by a slight amount and the navigational solution could be smoothed out from the sudden jumps, however addition of an IMU alone is not enough to negate the effects of the GPS outliers. Finally we demonstrate that the navigation solution derived from the m-estimator augmented sensor fusion graphs successfully mitigates the GPS outliers of this segment of the MAVs trajectory.

Figure 17 shows a plot of the entire trajectory of the MAV flying into the alley and then part way back.
CONCLUSIONS & FUTURE WORK

In conclusion, we proposed a graph based approach to sensor fusion of GPS and IMU measurements. We then augmented our sensor fusion graph with m-estimators in order to mitigate the effects of outliers upon our solution. We tested our algorithm on flight data collected from an AscTec Firefly MAV in a GPS-challenged environment and found that our approach successfully mitigated the effects of GPS outliers on the MAV’s navigation solutions. Through our approach, we are able to provide a more accurate, more frequent, and more robust MAV state estimate than using GPS alone.

REFERENCES


