Simultaneous State Estimation of UAV Trajectory Using Probabilistic Graph Models

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BIOGRAPHY

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ABSTRACT

For many airborne sensor applications, knowledge of the aerial platform's position and velocity assists in construction of the target image and mapping the resultant model onto a surveyed map. Since sensor data is processed during post-processing, emphasis is placed on an accurate trajectory estimate as opposed to a real-time navigation. In this paper we present a smoothing approach for integration of Inertial Measurement Unit (IMU) and Global Positioning System (GPS) pseudorange measurements in a tightly coupled UAV system. We combine all available sensor information in a probabilistic graph connected by a motion model that relates each state to its predecessor states. We then solve for a Maximum a Posteriori (MAP) estimate by finding the parameters that maximize the joint probability model derived from the probabilistic graph. An Iterative Least Squares (ILS) algorithm is used to solve for the parameters of the MAP estimate. In order to validate our algorithms, we set up an experimental test bed using an Asctec Pelican Quadrotor equipped a u-blox LEA-6T receiver. Implementing our algorithm on collected GPS Pseudorange and accelerometer flight data, we show that we can combine GPS code measurements and noisy IMU sensor data for smoother, more precise state estimates of a UAV's trajectory.

INTRODUCTION

Unmanned Air Vehicles (UAVs) are commonly used in Intelligence, Surveillance, and Reconnaissance (ISR) missions for their abilities to survey vast areas quickly, safely and efficiently. They can be equipped with a variety of sensors including imagers, radar, and SIGINT sensors and can gather in-depth information across a target region. Because there are many applications where knowledge of a UAV's position and velocity assists in reconstruction of sensor data, GPS measurements are often taken alongside sensor measurements. One example is ground feature reconstruction using airborne Synthetic Aperture Radar (SAR) Interferometry. Radar measurements taken from multiple positions are combined with precise GPS positions through post-processing to reconstruct geographical features [1]. GPS positions and velocities are also used in reconstruction of 3D models in geo-registered locations from motion video [2]. Traditionally, the goal of aerial platform state estimation is to estimate its current state and predict its future state at the next time step so that future state estimates can be corrected. However in order to complement the scenarios above, our goal is not only estimating and predicting our current and next states, but also rectifying our past estimates based on future measurements.

Navigational sensors can be categorized into external sensors and dead-reckoning sensors. External sensors such as GPS provide absolute positioning based on measurements to external landmarks or beacons. Dead-reckoning sensors, such as Inertial Navigation Systems, provide relative positioning based on one's previous position. Although more robust than external sensors, dead reckoning sensors can be subject to drift, accumulating large errors over time. Generally these sensor types are combined in the form of a GPS/INS system. Integration of these two sensors can be either loosely coupled or tightly coupled. Our method takes a tightly coupled approach to combine GPS and Inertial

Measurement Unit (IMU) measurements. Tightly coupled approaches allow for a more robust state estimate by incorporating IMU estimates directly into the GPS navigation solution.

Kalman type filters are commonly used for GPS/IMU sensor fusion, providing an iterative method to predict and update UAV state estimates for navigational purposes [3][4]. Since the focus of collected sensor data is in post-processed mapping and reconstruction of target images and not real-time navigation, we place an emphasis on estimating the entire trajectory of the UAV as opposed to a current state. We can leverage future state estimates and sensor measurement information to rectify predecessor state estimates. Thus a smoothing approach is preferred rather than a filtering approach towards localization. In prior work, Daellert and Kaess introduced a smoothing approach as a viable alternative to extended Kalman filters for positioning a robot's path through landmark measurements and a motion model [5]. Bryson et al. presented a smoothing approach to IMU, GPS, and monocular vision fusion for large-scale terrain reconstructions [6]. In this paper we present a smoothing approach for integrating measurements from low-cost GPS and IMU sensors with a simple UAV dynamics model.

APPROACH

The goal of our state estimation algorithm is to find the optimal estimated trajectory traversed by a UAV. By taking a smoothing approach, we allow sensor information from future states to propagate backwards and rectify previous state estimates. Our system incorporates GPS pseudorange code measurements tightly coupled with an Inertial Navigation System (INS) three-axis accelerometer. The UAV state being estimated at time i is denoted x_i and consists of UAV position, velocity and acceleration. We define the UAV dynamic model as a point particle moving in Earth-Centered Earth-Fixed (ECEF) component directions, x, y, and z at an estimated velocity and acceleration. As opposed to loosely coupled systems where INS measurements are integrated with position estimates output from a GPS receiver, our tightly coupled estimator combines accelerometer measurements in our position and clock bias calculation from the GPS pseudorange measurements [7]. The GPS and state calculations then directly complement the IMU measurements in estimating the IMU drift and bias in order to obtain more accurate acceleration measurements throughout the trajectory of the UAV.

To accomplish this smoothing and batch estimation of the UAV trajectory states, we first set up a probabilistic graph model to map the conditional dependencies between each state and measurement variable. This allows us to characterize the influence that measurements of a particular state have on its predecessor states. We then transform the probabilistic model into a maximum a posteriori probability estimation problem where we seek to find the most likely state estimates given specific sensor measurements from the GPS receiver and IMU accelerometer. We then solve the batch estimation problem through Iterative Least Squares (ILS) by minimizing each source of error and transitioning from one iterative state estimate to another through a the systems Jacobian functions.

Probabilistic Graph Model

We use a probabilistic graph, specifically a Bayes network to model random variables and their conditional dependencies in our system. A Bayes network is a directed acyclic graph where the set of nodes are connected by edges and each edge has a direction implying a conditional dependence. It is acyclic implying that there are no loops. Starting at any chosen node, it is impossible to follow a sequence of directed edges to eventually loop back to the initially chosen node.

Within our probabilistic graph for a GPS system, each node represents a sensor measurement or an object state. We designate four types of nodes, UAV states, satellite positions, GPS measurements, and IMU measurements. The edges within the GPS probabilistic graph represent the effect of a measurement upon an estimate of a state or motion between states. We model the UAV state estimates of each time step as nodes, connected throughout the trajectory timespan. Each node is linked to its predecessor through a single edge representing the propagated UAV state through its dynamics complemented by the IMU Accelerometer measurements. This applies a constraint to the probable positions of a state relative to its predecessor. Next, each GPS satellite visible across the timespan is designated as a different node. Each UAV state node then takes a GPS pseudorange measurement to each visible satellite at that particular measurement epoch.

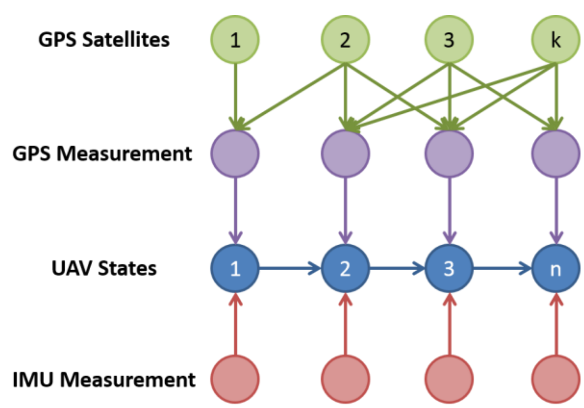

Figure 1: Visualization of GPS Probabalistic Graph

Model

From the modeled dependence structure, we form a joint probability model. The joint probability of the UAV trajectory is denoted as:

$$P(X,Z,A,dt,\beta)=P(x_0)\prod_{i=1}^{T}P(x_i\mid x_{i-1})\prod_{i=1}^{T}\prod_{k=1}^{K}P(z_i^k\mid x_i,\rho_i^k,dt)\prod_{i=1}^{T}P(\ddot{x}_i\mid\alpha_i,\beta)$$

Initial Position — Dynamics Model — GPS Code Measurements — IMU Accelerometer Measurements

where $P(X,Z,A,dt,\beta)$ is the joint probability of the trajectory for the set of all states X, set of all pseudorange measurements Z, and set of all accelerometer measurements A. $P(x_o)$ is a prior on the initialization state, stating certainty at the UAVs takeoff position. $P(x_i|x_{i-1})$ is the motion model propagated from previous states. $P(z_i^k|x_i,\rho_i^k,\mathrm{dt})$ is the GPS measurement model where the pseudorange to satellite k at time i, z_i^k, is parameterized by x_i, the observed pseudorange to satellite k at time i, ρ_i^k, and the receiver clock bias dt. $P(\ddot{x}_i|\alpha_i,\beta)$ is the IMU measurement model where $\ddot{x}_i$, the acceleration state is parameterized by the accelerometer measured accelerations a_i and the accelerometer biases β_i. T is the set of all time steps and K is the set of all visible satellites.

We define our target estimate trajectory states as the states and observations with maximum likelihood. By assuming that the system process and measurement models are Gaussian, we define the following

$$\begin{aligned} x_i &= f(x_{i-1})+w_i \\ z_i^k &= r(x_i,\rho_i^k,dt)+\varepsilon_i^k \\ \ddot{x}_i &= \alpha_i+\beta+v_i \end{aligned}$$

where f is the process model that defines the UAV's system dynamics, w_i is normally distributed zero-mean process noise with covariance Σ_D, r is the pseudorange measurement equation and ε_i^k is the pseudorange error with covariance Σ_ρ, $\alpha_i$is the measured acceleration, $\beta_i$is the accelerometer bias, and v_i is a zero-mean measurement noise with covariance Σ_α.

We then define the Gaussian probability distributions of the joint probability model as

$$P(x_i\mid x_{i-1})\propto\exp-\frac{1}{2}\left\|f(x_{i-1})-x_i\right\|_{\Sigma_D}^2$$

$$P(z_i^k\mid x_i,\rho_i^k,dt)\propto\exp-\frac{1}{2}\left\|r(x_i,\rho_i^k,dt)-z_i^k\right\|_{\Sigma_\rho}^2$$

$$P(\ddot{x}_i\mid\alpha_i,b_i)\propto\exp-\frac{1}{2}\left\|\ddot{x}_i-(\alpha_i+\beta+v_i)\right\|_{\Sigma_\alpha}^2$$

where $\|e\|_\Sigma^2$ is defined as the Mahalanobis distance of vector e with a covariance Σ. We define Θ as the collection of parameter variables (X,Z,A,dt,β) such that $\Theta\triangleq(X,Z,A,dt,\beta)$. We search for the parameters $\hat{\Theta}$ that will maximize our joint probability through a Maximum a Posteriori (MAP).

$$\hat{\Theta}=\arg\max_{\Theta}P(X,Z,A,dt,\beta)=\arg\min_{\Theta}-\log P(X,Z,A,dt,\beta)$$

By taking the negative log function of the probability maximizations, we transform the maximization problem into a minimization problem where the joint probability products become a series of sums.

$$\hat{\Theta}=\arg\min_{\Theta}\left\{\begin{aligned}&\sum_{i=1}^{T}\left\|f(x_{i-1})-x_i\right\|_{\Sigma_D}^2+\sum_{i=1}^{T}\sum_{k=1}^{K}\left\|r(x_i,\rho_i^k,dt)-z_i^k\right\|_{\Sigma_\rho}^2\\&+\sum_{i=1}^{T}\left\|\ddot{x}_i-(\alpha_i+\beta+v_i)\right\|_{\Sigma_\alpha}^2\end{aligned}\right\}$$

Iterative least squares is then used to numerically solve for the optimal variable estimate $\hat{\Theta}$.

Numerical Solution via Iterative Least Squares (ILS)

We rewrite the log likelihood of the joint probability as a sum of errors of their respective measurement and process models.

$$\hat{\Theta}=\arg\min_{\Theta}\left\{C_{Dynamics}+C_{PR}+C_{IMU}\right\}$$

C_D is the sum of all costs c_D attributed to the dynamics propagations of velocity and position from the predecessor state x_{i-1} to the estimated positions at the current state x_i. It has two components, the position propagation error and the velocity propagation error.

$$c_D=c_{D_{Position}}+c_{D_{Velocity}}=\left\|f(x_{i-1})-x_i\right\|_{\Sigma_D}^2$$

$$=\left\|\hat{x}_i-\left(\hat{x}_{i-1}+\hat{\dot{x}}_{i-1}\delta t+\frac{1}{2}\hat{\ddot{x}}_{i-1}(\delta t)^2\right)\right\|_{\Sigma_D}^2+\left\|\hat{\dot{x}}_i-\left(\hat{\dot{x}}_{i-1}+\hat{\ddot{x}}_{i-1}\delta t\right)\right\|_{\Sigma_D}^2$$

We denote the Jacobian for a single time step i for the dynamics costs as D_{Pi} and D_{Vi} respectively.

$$D_{Pi}=\begin{array}{c}\begin{array}{cccc}\quad x_{i-1}\quad & \quad\dot{x}_{i-1}\quad & \qquad\ddot{x}_{i-1}\qquad & \quad x_i\quad\end{array}\\ \begin{bmatrix}1&0&0&\delta t&0&0&.5(\delta t)^2&0&0&-1&0&0\\0&1&0&0&\delta t&0&0&.5(\delta t)^2&0&0&-1&0\\0&0&1&0&0&\delta t&0&0&.5(\delta t)^2&0&0&-1\end{bmatrix}\end{array}$$

$$D_{Vi}=\begin{array}{c}\begin{array}{cccc}\quad\dot{x}_{i-1}\quad & \quad\ddot{x}_{i-1}\quad & \quad x_i\quad & \quad\dot{x}_i\quad\end{array}\\ \begin{bmatrix}1&0&0&\delta t&0&0&0&0&0&-1&0&0\\0&1&0&0&\delta t&0&0&0&0&0&-1&0\\0&0&1&0&0&\delta t&0&0&0&0&0&-1\end{bmatrix}\end{array}$$

where δt is the calculation time step. D_{Pi} is the derivative of the position component of the dynamics process model and correlates the position of the i^{th} state with the i-1[th] state's position, velocity, and acceleration as defined in the dynamics model. D_{Vi} accomplishes the same except for the velocity.

C_{PR} is the sum of all costs c_{PR} attributed to the pseudorange measurement versus the geometric range between the estimated position and each satellite modified by the receiver clock bias.

$$c_{PR} = \left\| r\left(x_i, \rho_i^k, dt\right) - z_i^k \right\|_{\Sigma_\rho}^2 = \left\| \sqrt{\left(x^k - x_i\right)^2 + \left(y^k - y_i\right)^2 + \left(z^k - z_i\right)^2} - \left(\rho_i^k + cdt\right) \right\|_{\Sigma_\rho}^2$$

As per the standard ILS method for code-based navigation solution calculation, we denote the Jacobian for a single time step i for the Pseudorange costs as G_i, the geometry matrix each satellite-in-view k.

$$G_i = \begin{bmatrix} -\frac{\left(x^1 - x_i\right)}{\rho_i^1} & -\frac{\left(y^1 - y_i\right)}{\rho_i^1} & -\frac{\left(z^1 - z_i\right)}{\rho_i^1} & 1 \\ -\frac{\left(x^2 - x_i\right)}{\rho_i^2} & -\frac{\left(y^2 - y_i\right)}{\rho_i^2} & -\frac{\left(z^2 - z_i\right)}{\rho_i^2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -\frac{\left(x^k - x_i\right)}{\rho_i^k} & -\frac{\left(y^k - y_i\right)}{\rho_i^k} & -\frac{\left(z^k - z_i\right)}{\rho_i^k} & 1 \end{bmatrix}$$

The resultant geometry matrix influences the positional estimate as a given state i.

C_{IMU} is the sum of all costs c_{imu} attributed to the error between the IMU accelerometer measurements and the estimated accelerations.

$$c_{IMU} = \left\| \ddot{x}_i - \left(\alpha_i + \beta + v_i\right) \right\|_{\Sigma_\alpha}^2 = \left\| \hat{\ddot{x}}_i - \left(\alpha_i + \beta\right) \right\|_{\Sigma_\alpha}^2$$

We denote the Jacobian for a single time step i for the IMU error as Λ_i correlating the acceleration estimates at each time step to itself as well as the accelerometer biases.

$$\Lambda_i = \begin{bmatrix} \overbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}^{\ddot{x}_i} & \overbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}^{\beta} \end{bmatrix}$$

The errors have a covariance of Σ, the concatenated covariance matric of each error component.

$$\Sigma = \begin{bmatrix} \Sigma_{PR} & 0 & 0 \\ 0 & \Sigma_D & 0 \\ 0 & 0 & \Sigma_{IMU} \end{bmatrix}$$

The resultant Jacobian matrix J that relates the costs to each state variable is then the concatenation of each of the component Jacobians D_{Pi}, D_{Vi}, G_i, Λ_i at all time steps t=1 to T.

$$J = \begin{bmatrix} G_1 & & & & & & & & & & \\ & \ddots & & & & & & & & & \\ & & G_T & & & & & & & & \\ D_{P_1} & & & & & & & & & & \\ & \ddots & & & & & & & & & \\ & & D_{P_i} & & & & & & & & \\ & & & D_{V_1} & & & & & & & \\ & & & & \ddots & & & & & & \\ & & & & & D_{V_1} & & & & & \\ & & & & & & \Lambda_i & & & & \\ & & & & & & & \Lambda_i & & & \\ & & & & & & & & \Lambda_i & & \end{bmatrix}$$

For a 3-dimensional system of T time steps, we estimate 10 state variables at each time step, position, velocity, and accelerations $x_i, \ddot{x}_i, \ddot{x}_i$ along with the clock bias dt. We also estimate the system accelerometer bias in a single 3-dimensional variable β. As a result, the total number of estimate variables is 10 x T + 3.

From the dynamics process cost functions, we obtain 2 cost equations, position and velocity, for each 3-dimensional variable $x_i, \ddot{x}_i$ starting from the 2nd time step. This results in a 2 x 3 x (T-1) or 6 x (T-1) cost functions for the total system. The IMU error functions provide 3 cost functions per time step resulting in 3 x T functions. From the GPS pseudorange measurements, we obtain k_i cost functions depending on the satellites in view at each time step. Assuming 4 satellites are in view at each epoch, we obtain 4 x T cost functions.

The dimensions of the resultant Jacobian matrix are [13T – 6] x [10T + 3] resulting in an overdetermined system when 4 satellites are in view per time step. This also provides robustness available for state estimation at gap periods when satellite visibility drops below 4. Although a vast Jacobian matrix is created with components from each time step, the resultant matrix is largely a sparse matrix. We can leverage the sparseness of the Jacobian matrix in reducing the computation load and obtaining computationally effective batch estimation.

Afterwards ILS steps are used until the system converges towards optimal estimate for the states in the trajectory so far. The matrix H is the transition matrix that represents the ILS step towards optimization and minimization of the system costs.

$$H = \left(J^T \Sigma^{-1} J\right)^{-1} J^T \Sigma^{-1}$$
$$x_{iter+1} = x_{iter} + Hb$$

As each measurement is added on, past measurements are re-estimated and rectified. Through iteration, the optimal state estimates are achieved at the end of a trajectory where all pseudoranges and IMU data collected throughout the flight is used for a batch estimate for all states simultaneously.

EXPERIMENTAL RESULTS

To validate our algorithm, we used the Asctec Pelican Quadrotor as an experimental platform.

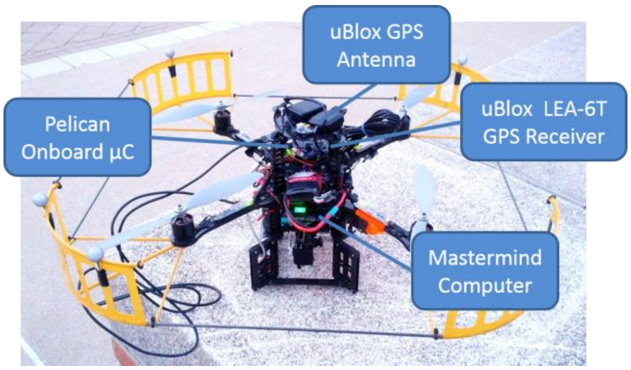

Figure 2: Experimental setup of an Asctec Pelican. It is equipped with an onboard IMU and an additional u-blox LEA-6T GPS receiver.

The Asctec Pelican has two on-board processors, a low level processor and a high level processor. The low level processor consists of an onboard microcontroller that is in charge of the flight controller of the Pelican. It contains an onboard INS consisting of IMU rate gyros and accelerometers. It also processes the different inner and outer loop controls for operating the quadrotor. The high level processor is an Asctec Mastermind board with an Intel® Core™ i7 processor. The Mastermind board serves as an on-board navigational computer for processing navigational algorithms. Connected to the Mastermind is a u-blox LEA-6T GPS receiver. For this experiment, flight sensor data from the IMU and the GPS receiver was collected and logged on the navigational computer for post-processing.

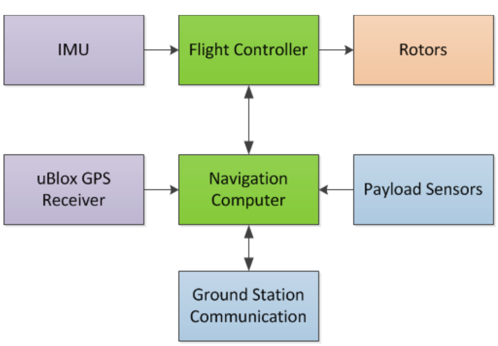

Figure 3: Command and Data Handling Diagram for the Asctec Pelican equipped with navigation equipment.

We implemented our algorithm on data collected from a sixty-one second flight. Figure 4 shows the resultant code-based navigation solution from the trajectory of the flight.

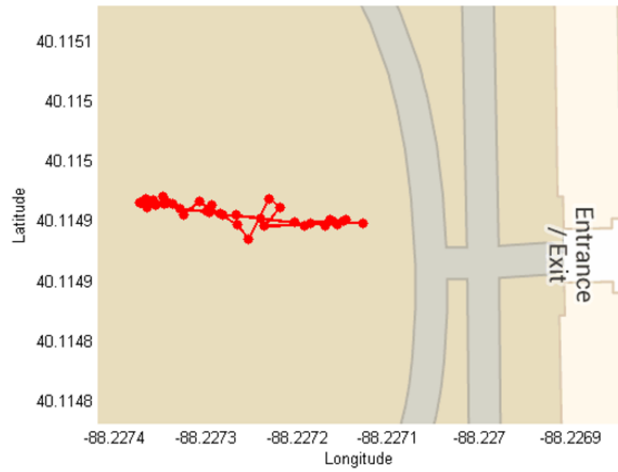

Figure 4: GPS only positioning of a UAV's trajectory.

In the test segment, GPS data was collected at a rate of 1 Hz and fused with filtered IMU data of the same rate. A 20° elevation angle mask constraint was placed on the available satellites and as a result each epoch had between 4-6 satellites visible.

From the different process and measurement models the resultant Jacobian of the flight is a 981 x 613 sparse matrix. The different segments of the Jacobian are marked as follows in Figure 5.

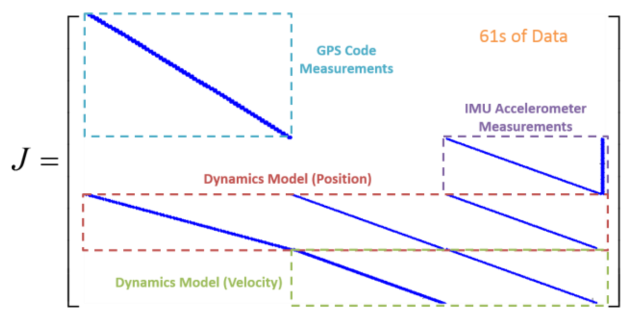

Figure 5: The nonzero elements of the resultant sparse Jacobian matrix.

Through the smoothing approach presented within this paper, we achieved a much more accurate UAV trajectory and positioning spread of less than 2 meters. The result is shown in Figure 6.

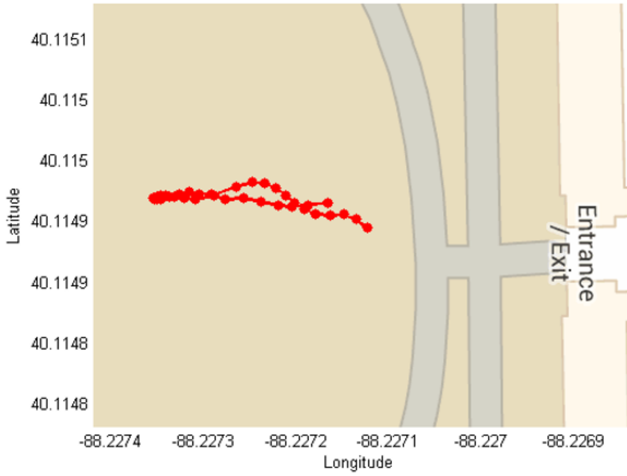

Figure 6: GPS with PGM estimated positioning of a UAV's Trajectory.

We then tested the effects of navigation under low satellite visibility. During aperiod where only four satellites were in view, we investigated the result of removing satellite PRN 22 on the navigation solution. As a result, the UAV position calculations during the period of time were degraded severely as shown in Figure 7. Positioning errors were upwards of 10 m during this period of low satellite visibility.

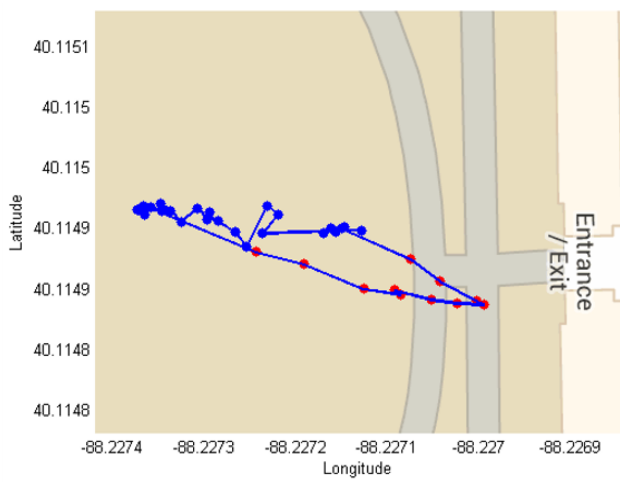

Figure 7: GPS navigation solution with a 10 s gap in satellite visibility

After applying our graph-based smoothing algorithm, we were able to contain the trajectory estimate within a 3 meter error as shown in Figure 8.

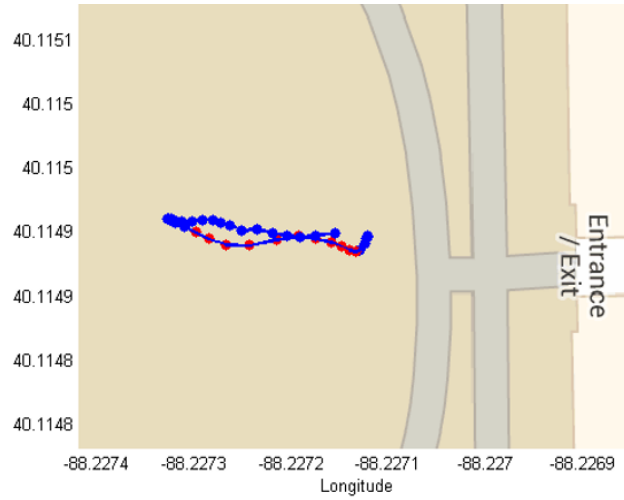

Figure 8: PGM improved GPS navigation solution with a 10 s gap in satellite visibility

CONCLUSION AND FUTURE WORK

In this paper we propose a smoothing approach for simultaneous estimation of a UAV's trajectory states. Our algorithm tightly integrates IMU accelerometer measurements with GPS pseudorange measurements using a probabilistic graph. We combine all sensor data taken throughout the flight in a single batch estimation process in order to estimate the full trajectory. We demonstrate that with our approach we are able to achieve a more accurate estimate of a UAV's trajectory even during situations where satellite visibility is momentarily lost. Future work will examine modifying the algorithm into an iterative version for real-time control and navigation of a UAV. We will also examine incorporation of different types of navigation sensors into our sensor fusion approach and the effects of leveraging payload sensor data on the navigation solution.

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