GPS Signal Authentication from Cooperative Peers

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Abstract—Secure, reliable position information is indispensable for many transportation systems and services, such as traffic monitoring, fleet management, electronic toll collection, route guidance, vehicle telematics, and emergency response. Unfortunately, civil Global Positioning System (GPS) signals are vulnerable to spoofing attacks. This paper introduces a signal authentication architecture based on a network of cooperative GPS receivers. A receiver in the network correlates its received military P(Y) signal with those received by other receivers (hereinafter referred to as cross-check receivers) so as to detect spoofing attacks. This paper describes three candidate structures to implement this architecture, and evaluates spoofing detection performance through theoretical analyses and field experiments. We show that the spoofing detection performance improves exponentially with increasing number of cross-check receivers. Even if the cross-check receivers are low-cost, unreliable, and in a challenging environment, cooperative authentication can match, if not outperform, a single, high-quality, reliable reference receiver in terms of spoofing detection performance.

Index Terms—Global Positioning System (GPS), global navigation satellite systems (GNSS), authentication, spoofing detection, cooperative, security, reliability

I. INTRODUCTION

Location awareness is crucial to many transportation systems and services including traffic monitoring, fleet management, electronic toll collection, route guidance, vehicle telematics, and emergency response [1]–[3]. The Global Positioning System (GPS) technology has been transforming the transportation landscape by allowing agencies to effectively monitor and manage transportation assets. In the area of road traffic monitoring, GPS data have significantly improved our ability to monitor traffic conditions in real time [4], [5]. Unlike dedicated traffic monitoring sensors installed in the pavement or along the roadside, GPS data can be collected very cheaply from personal navigation devices, GPS equipped smartphones, and from fleet vehicle monitoring systems. This has also introduced a new market based on buying large volumes of GPS data, processing it into useful traffic information, and finally selling the processed information for display on online maps or in navigation applications [6]. In addition to traffic monitoring, toll collection is also adopting GPS technology and benefits from GPS data. A notable example is the Toll Collect Project operated in Germany since 2005 [7], [8]. Using GPS to identify when a vehicle is on a tolled road, this system outperforms traditional toll gates in terms of wide-area coverage and flexible toll fee calculation [9].

Ever-growing adoption of GPS technology and dependence on GPS data call for techniques capable to authenticate GPS signals so as to provide secure, reliable location information. Unfortunately, security was not an initial design consideration for civil use of GPS [10]. The power of GPS signals received on the Earth is as low as $10^{-16}$ W, even below the thermal noise floor [11]. The civil signals are unencrypted, with their structures explicitly described in publicly-available documents [12]. As a result, civil GPS receivers are vulnerable to attacks such as jamming, meaconing, and spoofing [13]–[17].

Jamming is the intentional broadcast of a high power interfering signal at the GPS frequency in order to deny GPS receivers within a certain area access to the GPS signals. Hence, jamming is disruptive but usually detected by the receiver whenever it stops tracking satellites.

Mecaconing, as a kind of replay attack, is the recording and rebroadcast of GPS signals that overpower the authentic signals. A meaconing attack that replays the whole GPS spectrum can even fool a military receiver. However, an inherent limitation of meaconing is that the position calculated by a compromised receiver is equal to the position of the attacker’s antenna used to record the GPS signals. Hence, meaconing can expose the attacker’s position, and manipulation of the position solution is subject to the physical maneuverability of the attacker’s antenna.

Spoofing is a much more sophisticated and dangerous attack than jamming or meaconing. A spoofer synthesizes and broadcasts counterfeit GPS signals in order to manipulate a target receiver’s reported position or time, or both [13], [18]. In comparison to jamming and meaconing, spoofing poses a greater security risk because it is covert and it can manipulate a target receiver’s output at the attacker’s will. There has been an experiment showing that a spoofer can mislead a GPS-directed semi-autonomous vehicle without trigging any alarms [19].

An even more troublesome scenario is self-spoofing. For example, a GPS data vendor may mix authentic GPS data with faked data, and profit from selling such kind of mixed data. A driver may spoof the GPS receiver in his vehicle’s monitoring system in order to avoid paying a toll. In this paper, we aim at techniques that do not only protect receivers from being spoofed, but also protect a third party from counterfeit GPS data.

A. GPS spoofing countermeasures

So far, a variety of methods have been proposed to harden civil GPS receivers against spoofing attacks. These methods can be generally categorized into three groups: external assistance, signal statistics, and cryptographic authentication. The first group performs consistency checks against metrics external to the GPS subsystem, such as the information

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from inertial sensors, odometers, cellular networks, and high-stability clocks [20], [21]. The second group performs statistical tests on features inherent in GPS signals, including angle of arrival [22], [23], signal quality [24], signal power [25], [26], and multipath [27]. The third group relies on unpredictable cryptographic information carried by GPS signals [10], [28]–[30]. Unlike the first group of methods, cryptographic methods do not require any additional hardware, which can be a hurdle to mass-market GPS applications that are sensitive to cost, weight, and/or size. In comparison to the second group, cryptographic methods enable a receiver to differentiate authentic signals from counterfeit signals with higher confidence and robustness, especially when the receiver is moving so that the statistics of authentic signals can be highly unstable.

Three types of cryptographic spoofing countermeasures have been explored in recent literature. The first option, known as navigation message authentication (NMA), inserts public-key digital signature into the navigation message [28], [30]–[32]. Another strategy is to interleave spread spectrum security codes (SSSC) with normal civil GPS spreading codes so that parts of spreading sequences are periodically unpredictable [28], [33]. Both NMA and SSSC require significant modifications to the legacy GPS signal structure. They are unlikely to be implemented in the coming decade due to the static nature of GPS interface specification (IS) and long deployment cycles. The third approach relies on codeless cross-correlation of unpredictable encrypted military P(Y) code between two civil GPS receivers [10], [29], [34], [35]. Without any modification to the GPS IS, this approach is practical today. Furthermore, the cross-correlation method can easily enable a third party, such as a traffic data vendor or a Location Assurance Provider [36], to ensure that an asserted position is bona fide.

The cross-correlation spoofing detection method borrows the idea from the dual-frequency GPS codeless receiver, which correlates the L1 and L2 P(Y) codes in order to find the differential delay between the phases of two codes [37]. As shown in Fig. 1 this method correlates a snippet of L1 signal from the receivers to be authenticated (hereafter referred to as “user receivers”) with a snippet from the reference receiver. Both snippets are known to contain the same part of the military P(Y) code broadcast by a GPS satellite visible to both receivers. Although the P(Y) code is encrypted and thus unknown, and although its received versions are noisy and may be distorted by a narrow-band RF front-end [29], when conducting cross-correlation, the P(Y) code components in the two snippets are sufficiently similar to create a high correlation peak if neither the user receiver nor the reference receiver is spoofed. However, if the reference receiver is also spoofed, especially by the same spoofer to the user receiver, the authentication result will be incorrect. Previous papers [10], [29] have analyzed the performance of the cross-correlation spoofing detection method using one reliable reference receiver. In addition, they proposed employing a few dedicated reference stations to provide GPS signal authentication service for a wide area. Despite the strong merits, such a client-server authentication service has some limitations. First and foremost, it requires considerable investment into the setup of reference stations, not to mention the maintenance cost. Second, since fixed reference stations can be located, they are vulnerable to organized, targeted jamming and spoofing attacks, and loss of a majority of the reference stations may paralyze the authentication service.

B. Authentication from multiple cooperative peers

In this paper, we extend the dual-receiver P(Y)-code correlation method to a network of receivers, and present a GPS signal authentication architecture in an ad hoc, cooperative manner. The fundamental difference from the client-server approach [10], [29] is that our architecture relies on multiple voluntary peers (hereinafter referred to as “ad-hoc cross-check receivers” or simply “cross-check receivers”) as references. The cross-check receivers can be mobile, low-quality, unreliable, and even spoofed. The authentication process consists of two steps: pair-wise check and decision aggregation. In the pair-wise check, the P(Y) signal received by a user receiver is correlated with that received by each cross-check receiver. Each such correlation provides a decision as to the authenticity of the signal received by the user receiver. In decision aggregation, the pair-wise decisions are aggregated to determine if the user receiver is spoofed.

The cooperative manner is superior to the client-server manner in terms of cost, user capacity, and robustness, thanks to unlimited geographically-dispersed low-cost ad-hoc cross-check receivers. However, one should be aware that an ad-hoc cross-check receiver is less reliable than a dedicated reference receiver. First, a mass-market GPS receiver, especially one embedded in a smartphone, may not be as good as a dedicated geodetic-grade receiver in terms of the antenna and the signal conditioning circuit. Second, a cross-check receiver may intentionally be malicious so that it provides no or even negative contribution to the final authentication result. Third, a cross-check receiver can also be spoofed, and sometimes a user receiver and a cross-check receiver may be spoofed by the same spoofer if they are not sufficiently far apart. We shall further show in this paper that our proposed architecture is actually robust against these potential issues because the
spooing detection performance improves exponentially with increasing number of cross-check receivers.

C. Organization of the remainder of this paper

Section II describes three candidate structures to implement our proposed cooperative authentication architecture, and compares their advantages and disadvantages. Section III presents a probabilistic analysis of authentication performance under the assumption that cross-check receivers can be spoofed or malicious with certain probabilities. Section IV validates the theoretical conclusions through a few numerical examples. Section V shows field experiment results on pair-wise check performance. Finally, Section VI concludes this paper.

II. Authentication System Structures

There are multiple approaches to implementing our proposed cooperative authentication system. These approaches differ from one another mainly in where correlations are computed. One approach is to distribute correlation computation to cross-check receivers. Another option is to compute all the correlations in a centralized way, either by the user receiver itself or by a third party which wants to ensure the validity of the position and clock reported by the user receiver. In this section, we present three candidate structures, and qualitatively discuss their tradeoffs between authentication delay, cost, CPU time, and robustness.

This paper considers two purposes of GPS signal authentication: spoofing detection and position assertion verification. The first purpose is concerned with the scenario that a user receiver wants to check the authenticity of its received signals. Since a successful spoofing attack usually needs to synthesize the GPS signals of all the satellites in view [13], [18], checking the authenticity of the signal from one satellite suffices to detect a spoofing attack. To this end, the user receiver and all cross-check receivers only need to collect a snippet of quadrature-phase baseband signal for one GPS satellite visible to all of them. The second purpose is concerned with the scenario that a third party (e.g., a fleet manager) to check whether a position asserted by a user receiver is authentic or not. To enable position assertion verification, the user receiver must report a snippet that contains the complex baseband signal (both in-phase and quadrature). Then, the third party can track multiple satellites from the snippet and calculate the position solution, which ought to match the asserted position. In addition, the third party can extract the quadrature-phase baseband signal for a GPS satellite from the snippet, and correlate it with the quadrature-phase baseband snippets from cross-check receivers. The correlation results are used to determine the authenticity of the user reported snippet. All the following three candidate structures will achieve the first purpose, while only the last one is designed to support the second purpose.

A. Candidate Structure 1: Correlation computed by cross-check receivers

Figure 2 illustrates the first candidate structure, in which correlation computation is distributed to cross-check receivers.
user receiver aggregates the decisions from the $N$ reference receivers, and determines the authenticity of its received signal by an appropriate statistical measure. Since snippets of GPS signals have to be transported over a communication network, a security protocol, such as TLS and IPsec [38], should be used to avoid man-in-the-middle attacks.

The authentication process can be performed in near real-time, and the time delay mainly depends on data collection, communication, and computation. According to Psiaki et al. [29], a snippet of approximate 1 second is generally needed for reliable spoofing detection. A narrow-band GPS front-end usually has a bandwidth of 2.4 MHz, and 1-second 1-bit quadrature-phase samples yield 2.4 M bits of data. For current 3G/4G cellular networks, it typically takes 1 second or less to transfer one snippet. The time of computation depends, but a rule of thumb is that a receiver must have the capability of processing 1-second data within 1 second. Since the time for sending and responding requests and aggregating decisions is usually negligible, the authentication process can take as short as $2 + N$ seconds: 1 second for collecting snippets, $N$ seconds for transferring the user receiver’s snippet to $N$ cross-check receivers, and 1 second for computing the correlations. It is worth noting that our cooperative authentication does not require highly reliable spoofing detection for each cross-check receiver, and thus allows a much shorter snippet to be collected. Therefore, a delay of $2 + N$ seconds is a conservative estimate. Besides, if the user receiver can upload its snippet to a cloud service for file-sharing, from which the cross-check receivers can download the snippet simultaneously, then the authentication delay can be shortened to 4 seconds: 1 second for collecting snippets, 1 second for uploading, 1 second for downloading, and 1 second for computing the correlations.

An obvious advantage of this structure is no requirement of external support (assuming that a file-sharing cloud is not used). However, unlike the other three candidate structures to be described, this structure requires each cross-check receiver to compute a correlation using its own computation power. The CPU time consumption is generally acceptable because one cross-check receiver only computes one correlation (compared to the second structure where the user receiver needs to compute $N$ correlations). In practice, the cross-check receivers with more spare CPU time will more likely respond to the authentication request.

As mentioned in Section II-B an issue with cooperative authentication is that there may exist some spam receivers being deliberately malicious (or playfully mischievous). In this structure, a malicious cross-check receiver may reply to the user receiver with a random decision independent of the correlation, or even worse, a decision always opposite to the correct decision based on the correlation. In Section III we shall show that the performance deterioration due to malicious cross-check receivers can be compensated by more cross-check receivers.

B. Candidate Structure 2: Correlation computed by the user receiver

Figure 3 illustrates the second candidate structure in which correlation computation is centralized to the user receiver. The major difference from the first candidate structure is that after the user receiver and cross-check receivers collect snippets, the cross-check receivers send their snippets to the user receiver. The user receiver computes $N$ correlations, based upon which it determines whether its received signal is authentic or not.

In this structure, the user receiver has to receive $N$ snippets and then compute $N$ correlations. If we still assume that it takes 1 second to transfer a snippet or to compute a correlation, the whole authentication process will take $1 + 2N$ seconds, a much longer delay in comparison to the first and second candidate structures.

The biggest advantage of this structure is that the user receiver can operate in a status close to radio silence because it does not send its snippet to any cross-check receiver or third party. Therefore, this structure is suitable for scenarios such as an on-duty drone authenticating its received GPS signals.

Another advantage with this structure is its better resistance to malicious cross-check receivers because the only way to disturb the authentication process is to send a random, irrelevant snippet. In Section III, we shall show that such kind of disturbance causes less performance deterioration than a cross-check receiver that always provides the incorrect decision.

For commercial applications, this structure saves the CPU time of cross-check receivers. Thus, more receivers are willing to respond to the authentication request.

C. Candidate Structure 3: Correlation computed by a third party

Figure 4 illustrates the third candidate structure, in which a trusted third party is in charge of collecting snippets, computing correlations, and aggregating decisions. Unlike the first and second structures, in addition to letting the user receiver know whether its received GPS signal is authentic or not, this structure enables a third party (which is usually
TABLE II
COMPARISON OF THE THREE CANDIDATE STRUCTURES

<table>
<thead>
<tr>
<th>Candidate Structure</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>External support required</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Authentication delay (seconds)‡</td>
<td>$2 + N^\dagger$</td>
<td>$1 + 2N$</td>
<td>2</td>
</tr>
<tr>
<td>User receiver CPU time</td>
<td>tiny</td>
<td>huge</td>
<td>no</td>
</tr>
<tr>
<td>Cross-check receiver CPU time</td>
<td>huge</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>User receiver sends snippet out</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Cross-check receiver sends snippet out</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Allow a third party to verify user receiver’s assertion</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

† Assume that it takes one second for a receiver to transfer a snippet or to compute a correlation, and it takes negligible time for a third party to compute a correlation.

‡ Four seconds if a file-sharing service is used for cross-check receivers to download a snippet simultaneously.

III. ANALYSIS OF AUTHENTICATION PERFORMANCE

Authentication is essentially a statistical hypothesis test, so it has a probability of making two types of errors: false alarm and missed detection. This section is devoted to a rigorous analysis of the probability of the two types of errors in cooperative authentication.

A. Assumptions and notations

In order to simplify the analysis, we assume that all ad-hoc cross-check receivers have the same spoofing detection performance, namely, the same probability of false alarm and the same probability of missed detection. A cross-check receiver can be malicious with a certain probability. Additionally, a cross-check receiver can be spoofed with a certain probability, and the spoofer can be the same as or different from the spoofer which is attacking the user receiver. The list below summarizes the notations used throughout this paper.

$A$ Final authentication result from aggregating all $A_i$, $i = 1, \ldots, N$.

$A_i$ Pair-wise check decision using the $i$th cross-check receiver, $i = 1, \ldots, N$: $A_i = 0$ “authentic”, and $A_i = 1$ “spoofed”.

$\alpha$ Equal to $\text{Prob}(A_i = 1|S = 0)$, for all $i = 1, \ldots, N$, probability of false alarm using an unspoofed, nonmalicious cross-check receiver.

$\beta$ Equal to $\text{Prob}(A_i = 0|S = 1)$, for all $i = 1, \ldots, N$, probability of missed detection using an unspoofed, nonmalicious cross-check receiver.

$C$ Pair-wise check test statistic.

$F(x; n, p)$ Cumulative distribution function (CDF) of a binomial random variable $X$ with parameters $n$ and $p$.

$H_0$ Null hypothesis that a user receiver’s snippet and a cross-check receiver’s snippet contain the same $P(Y)$ code.

$H_1$ Alternative hypothesis that a user receiver’s snippet and a cross-check receiver’s snippet contain different $P(Y)$ codes.

D. Comparison of the three structures

Table II compares the advantages and disadvantages of the three candidate structures. It can be seen that an external support, such as a third party in charge of the whole authentication process or a cloud for computing correlations, can greatly reduce authentication delay and offload the intensive computations. In addition, an external support can help find cross-check receivers by maintaining a continuously-updated database of available receivers. An external support can also help mitigate the negative effect due to malicious cross-check receivers by maintaining a database of historical performance of cross-check receivers. Therefore, for most commercial GPS authentication systems, the third structure should be used to exploit the benefit from an external support. Nevertheless, for sceneries where an external support is impossible or undesirable, the first and second structures still have their merits.
Number of cross-check receivers. 
Normal distribution with mean $\mu$ and variance $\sigma^2$.

Equal to $\text{Prob}(A = 1 | S = 0)$, probability of false alarm of the final authentication result.

Equal to $\text{Prob}(A = 0 | S = 1)$, probability of missed detection of the final authentication result.

Equal to $1 - P_{MD}$, probability of detection, also referred to as detection power.

Probability of (a) a cross-check receiver being spoofed by the same spoofing attacker that is attacking the user receiver and (b) a cross-check receiver being malicious such that its pair-wise check decision is always opposite to the correct decision based on the correlation.

Probability of (a) a cross-check receiver being spoofed by a different spoofing attacker that is attacking the user receiver and (b) a cross-check receiver being malicious such that its pair-wise check decision is based on the correlation involving a random, irrelevant snippet.

True status of user receiver: $S = 0$ “authentic,” and $S = 1$ “spoofed.”

Decision aggregation test statistic, equal to $\sum_{i=1}^{N} A_i$, number of “spoofed” decisions.

Decision aggregation spoofing detection threshold. The user receiver is determined to be “authentic” if $X < \xi$ and to be “spoofed” if $X \geq \xi$.

Pair-wise check decision threshold. If $C \geq \zeta$ then $H_0$ will be accepted, otherwise $H_1$ will be accepted.

**B. Signal model and performance of pair-wise check**

In this subsection, let Receiver 1 be a user receiver, and Receiver 2 be an ad-hoc cross-check receiver. Suppose that both receivers track the GPS L1 signal with perfect carrier and symbol timing recovery. The quadrature-phase baseband signals that contain the L1 P(Y) code are given by

$$s_1[t] = \Lambda_1 p_1[t] + n_1[t],$$

$$s_2[t] = \Lambda_2 p_2[t] + n_2[t],$$

where $t \in \{1, 2, \ldots, T\}$ is the index of a total of $T$ samples, $\Lambda_1$ and $\Lambda_2$ are the received P(Y) code amplitudes (after distortion and attenuation) for the two receivers, $p_1[t]$ and $p_2[t] = \pm 1$ denote the known P(Y) code sequences, and $n_1[t] \sim \mathcal{N}(0, \sigma_1^2)$ and $n_2[t] \sim \mathcal{N}(0, \sigma_2^2)$ account for receiver noises and other irrelevant GPS signals. The spoofing detection is based on the test statistic

$$C = \frac{1}{T} \sum_{t=1}^{T} s_1[t] s_2[t].$$

Define $c[t] = s_1[t] s_2[t]$ for all $t \in \{1, 2, \ldots, T\}$. Under the hypothesis $H_0$ that both receivers receive the same P(Y) code, i.e., $p_1[t] = p_2[t]$ for all $t$, the expectation and variance of $c[t]$ are given by

$$E(c[t]) = E((\Lambda_1 p_1[t] + n_1[t])(\Lambda_2 p_2[t] + n_2[t]))$$

$$= \Lambda_1 \Lambda_2;$$

$$\text{Var}(c[t]) = E((\Lambda_1 p_1[t] + n_1[t])^2(\Lambda_2 p_2[t] + n_2[t])^2)$$

$$= (E(c[t]))^2,$$

$$= \Lambda_1^2 \sigma_1^2 + \Lambda_2^2 \sigma_2^2 + \sigma_1^2 \sigma_2^2.$$

By the central limit theorem (CLT), for a very large $T$, we have

$$C_{H_0} \sim \mathcal{N}(\mu_{H_0}, \sigma_{H_0}^2) = \mathcal{N}\left(\Lambda_1 \Lambda_2, \frac{\Lambda_1^2 \sigma_1^2 + \Lambda_2^2 \sigma_2^2 + \sigma_1^2 \sigma_2^2}{T}\right).$$

Under the hypothesis $H_1$ that the two receivers receive different P(Y) codes, let us assume that $p_1[t]$ is independent from $p_2[t]$ for all $t$. Then, the expectation and variance of $c[t]$ are given by

$$E(c[t]) = E((\Lambda_1 p_1[t] + n_1[t])(\Lambda_2 p_2[t] + n_2[t])) = 0;$$

$$\text{Var}(c[t]) = E((\Lambda_1 p_1[t] + n_1[t])^2(\Lambda_2 p_2[t] + n_2[t])^2)$$

$$= (E(c[t]))^2,$$

$$= (\Lambda_1^2 + \sigma_1^2)(\Lambda_2^2 + \sigma_2^2).$$

By CLT, for a very large $T$, we have

$$C_{H_1} \sim \mathcal{N}(\mu_{H_1}, \sigma_{H_1}^2) = \mathcal{N}\left(0, \frac{\Lambda_1^2 + \sigma_1^2)(\Lambda_2^2 + \sigma_2^2)}{T}\right).$$

The signal-to-noise ratio (SNR) for the received signals are given by $\gamma_1 = \Lambda_1^2/\sigma_1^2$ and $\gamma_2 = \Lambda_2^2/\sigma_2^2$. Normalizing $\gamma_1$ by $\sigma_1$ and $\gamma_2$ by $\sigma_2$, and considering the fact that $\gamma_1 \ll 1$ and $\gamma_2 \ll 1, \gamma_1 + \gamma_2 < 1$, we can finally simplify (6) and (9) into

$$C_{H_0} \sim \mathcal{N}\left(\sqrt{\gamma_1 \gamma_2}, \frac{\gamma_1 + \gamma_2 + 1}{T}\right) \approx \mathcal{N}(0, 1/T).$$

Given a pair-wise check decision threshold $\zeta$, if $C \geq \zeta$ then the null hypothesis $H_0$ will be accepted, otherwise the alternative hypothesis $H_1$ will be accepted. Thus, the probability of false alarm $\alpha$ and the probability of missed detection $\beta$ are given by

$$\alpha = Q((\sqrt{\gamma_1 \gamma_2} - \zeta)\sqrt{T}),$$

$$\beta = Q(\zeta \sqrt{T}),$$

where the Q-function $Q(x) = (2\pi)^{-1/2} \int_{x}^{\infty} \exp(-u^2/2) \, du$ is the tail probability of the standard normal distribution. Fig. 5 shows the receiver operating characteristic (ROC) curves under the following settings:

- User receiver carrier-to-noise ratio ($C/N_0$): fixed to 38 dB;
- Cross-check receiver $C/N_0$: varying from 36 dB to 41 dB;
- Noise equivalent bandwidth: 2.4 MHz;
- Number of samples in a snippet ($T$): 2.4 $\times$ 10$^6$.

A ROC curve connects $(\alpha, 1 - \beta)$ pairs for different thresholds $\zeta$. In general, the closer a ROC curve is to the top left corner (which represents an ideal spoofing detector that detects all
spoofing attacks without issuing any false alarms), the better pair-wise check performance is.

The Chernoff bound of Q-function is $Q(x) \leq \frac{1}{2} \exp\left(-x^2/2\right)$ for all $x > 0$. When the threshold $\zeta$ is chosen properly, i.e., $0 < \zeta < \sqrt{T\gamma_2}$, increasing $T$ decreases both $\alpha$ and $\beta$ exponentially, as shown by

$$\alpha \leq \frac{1}{2} \exp\left(-\left(\sqrt{T\gamma_2} - \zeta\right)^2 T\right),$$

$$\beta \leq \frac{1}{2} \exp\left(-\zeta^2 T\right).$$

The above upper bounds on spoofing detection errors are based on a single pair-wise check. For $N$ cross-check receivers, the total number of samples increases to $NT$. Therefore, we can conjecture that the probability of false alarm and missed detection will both decrease exponentially with increase of $N$.

In the following two subsections, we show that this conjecture is true, even though the cross-check receivers can be spoofed or malicious.

C. Channel models

Since both $S$ and $A_i$ are binary, spoofing detection can be considered as an asymmetric communication channel. When the $i$th cross-check receiver is not spoofed or malicious, the channel model is simply given by the following.

$$S = 0 \quad 1 - \alpha \quad A_i = 0$$

$$S = 1 \quad \beta \quad A_i = 1$$

When the $i$th cross-check receiver is spoofed by a different spoofer to the user receiver or the cross-check receiver behaves in a malicious manner such that its authentication decision is based on the correlation involving a random, irrelevant snippet, the snippets from two receivers do not match whether the user receiver is spoofed or not. Therefore, the channel model is given by the following.

$$S = 0 \quad \beta \quad A_i = 0$$

$$S = 1 \quad 1 - \beta \quad A_i = 1$$

When the $i$th cross-check receiver is spoofed by the same spoofer to the user receiver or the cross-check receiver purposely responds with an authentication decision always opposite to the correct decision based on the correlation, the channel becomes the following.

$$S = 0 \quad 1 - \tilde{\alpha} \quad A_i = 0$$

$$S = 1 \quad \tilde{\beta} \quad A_i = 1$$

Among the above three channel models, the first occurs with a probability $1 - P_{SD} - P_{SS}$, the second occurs with a probability $P_{SD}$, and the third occurs with a probability $P_{SS}$. Therefore, the aggregated channel is given by the following, where

$$\tilde{\alpha} = (1 - P_{SD} - P_{SS})\alpha + (P_{SS} + P_{SD})(1 - \beta),$$

$$\tilde{\beta} = (1 - P_{SS})\beta + (P_{SS})(1 - \alpha).$$

D. Final authentication performance after aggregating decisions

Let $X = \sum_{i=1}^{N} A_i$ and $\xi$ be a preset threshold, where $\xi$ is an integer such that $0 \leq \xi \leq N$. The user receiver is determined to be “authentic” if $X < \xi$ and to be “spoofed” if $X \geq \xi$. Thus, we have

$$P_{FA} = \Prob(A = 1|S = 0) = \Prob(X \geq \xi|S = 0)$$

$$= \sum_{m=\xi}^{N} \left(\begin{array}{c} N \\ m \end{array}\right) \tilde{\alpha}^m (1 - \tilde{\alpha})^{N-m},$$

$$P_D = \Prob(A = 1|S = 1) = \Prob(X \geq \xi|S = 1)$$

$$= \sum_{m=\xi}^{N} \left(\begin{array}{c} N \\ m \end{array}\right) (1 - \tilde{\beta})^m \tilde{\beta}^{N-m}.$$
Equations (16) to (19) show that \( P_{SD} \) only affects \( P_{FA} \), while \( P_{SS} \) affects both \( P_{FA} \) and \( P_D \). Therefore, we can expect that \( P_{SS} \) deteriorates performance more significantly than \( P_{SD} \) does.

The CDF of a binomial random variable \( Y \) with parameters \( n \) and \( p \) can be expressed as

\[
F(y; n, p) = \sum_{m=0}^{\lfloor y \rfloor} \binom{n}{m} p^m (1-p)^{n-m},
\]

where \( \lfloor y \rfloor \) is the greatest integer less than or equal to \( y \). When \( y \leq np \), by Hoeffding’s inequality [40], an upper bound is given by

\[
F(y; n, p) \leq \exp\left(-2\frac{(np-y)^2}{n}\right).
\]

Rewrite (18) and (19) as \( P_{FA} = F(N - \xi; N, 1 - \tilde{\alpha}) \) and \( P_{MD} = 1 - P_D = F(\xi - 1; N, 1 - \tilde{\beta}) \). Considering a threshold selection strategy \( \xi = \kappa N \) such that

\[
N\tilde{\alpha} \leq \xi = \kappa N \leq N(1 - \tilde{\beta}),
\]

we have

\[
P_{FA} \leq \exp\left(-2\frac{(\xi - \tilde{\alpha}N)^2}{N}\right),
\]

\[
P_{MD} \leq \exp\left(-2\frac{(N(1 - \tilde{\beta}) - \xi)^2}{N}\right).
\]

It can be seen that both \( P_{FA} \) and \( P_{MD} \) decrease exponentially with increase of \( N \). The parameter \( \kappa \) determines how fast \( P_{FA} \) and \( P_{MD} \) shrink. A larger \( \kappa \) hastens exponential decay of \( P_{FA} \), while a smaller \( \kappa \) hastens exponential decay of \( P_{MD} \).

In addition, (22) implies a fundamental requirement

\[
\tilde{\alpha} + \tilde{\beta} < 1;
\]

unless the requirement was met, increasing \( N \) would not improve authentication performance.

E. Impact of spoofed or malicious cross-check receivers

In (23) and (24), if we choose \( \kappa = \frac{1}{2}(1 + \tilde{\alpha} - \tilde{\beta}) \), both \( P_{FA} \) and \( P_{MD} \) decrease at the same rate, on the order of \( \exp(-N(1 - \tilde{\alpha} - \tilde{\beta})^2) \). Therefore, the parameter \( \lambda = 1 - \tilde{\alpha} - \tilde{\beta} \) is a figure of merit characterizing how fast the final authentication performance improves with an increasing \( N \). By (16) and (17), we have

\[
\lambda = 1 - \tilde{\alpha} - \tilde{\beta} = (1 - \alpha - \beta)(1 - 2P_{SS} - P_{SD}),
\]

which indicates that the factor \( 1 - 2P_{SS} - P_{SD} \) is the penalty for the unreliability of ad-hoc cross-check receivers.

Equation (26) shows that \( P_{SS} \) causes twice as great performance deterioration as \( P_{SD} \) does. \( P_{SS} \) is the probability of two events: (a) a cross-check receiver being spoofed by the same spoofer that is attacking the user receiver, and (b) a cross-check receiver being malicious such that its authentication decision is always opposite to the correct decision based on the correlation. In practice, it is recommended to choose a cross-check receiver at least hundreds of meters away from the user receiver in order to reduce the probability of Event (a). Event (b) can only happen in Candidate Structure 1, so \( P_{SS} \) can be assumed to be zero for Candidate Structures 2 to 4. Furthermore, if information about the historical performance of cross-check receivers is available, some iterative learning algorithms [41] can be used to identify malicious cross-check receivers and preclude their negative impacts.

IV. Numerical Examples

The previous section has shown an exponential decay of \( P_{FA} \) and \( P_{MD} \) with increasing number of cross-check receivers. Furthermore, (26) shows that the performance deterioration due to \( P_{SS} \) is twice as great as that due to \( P_{SD} \). These theoretical conclusions are based on the upper bounds given by (23) and (24). This section presents several numerical results computed using (18) and (19) for the purpose of validating the theoretical conclusions.

According to Fig. 5, we assume the following performance of the pair-wise check throughout this section:

- \( \alpha = 0.001 \) and \( \beta = 0.15 \) (corresponding to \( C/N_0 \approx 38.1 \) dB) for a reliable, low-quality cross-check receiver, and
- \( \alpha = 0.0001 \) and \( \beta = 0.05 \) (corresponding to \( C/N_0 \approx 40.4 \) dB) for a reliable, high-quality reference receiver.

A. Receiver operating characteristic (ROC) curves

By (18) and (19), for fixed \( \tilde{\alpha} \), \( \tilde{\beta} \), and \( N \), the final authentication performance varies at various threshold settings of \( \xi \). Since \( X = \sum_{i=1}^{N} A_i \) is always an integer between 0 and \( N \), varying \( \xi \) results in \( N + 1 \) discrete pairs of \( P_{FA} \) and \( P_D \). Therefore, a ROC curve is a piecewise linear curve connecting the \( N + 1 \) points.

Figure 6 shows the ROC curves for two cases: all cross-check receivers are reliable (\( P_{SS} = P_{SD} = 0 \)); cross-check receivers can be spoofed or malicious with probabilities \( P_{SS} = 0.1 \) and \( P_{SD} = 0.1 \). It can be seen that increasing number of cross-check receivers always improves performance. When cross-check receivers are unreliable with such a large probability, four unreliable cross-check receivers are sufficient to match the performance of a single reliable, low-quality cross-check receiver (\( P_{FA} = 0.001 \) and \( P_{MD} = 0.15 \)), and seven can match a single reliable, high-quality reference receiver (\( P_{FA} = 0.0001 \) and \( P_{MD} = 0.05 \)).

B. Exponential decay of \( P_{FA} \) and \( P_{MD} \) with increasing \( N \)

Figures 7 and 8 show probability of missed detection and probability of false alarm, both as functions of number of cross-check receivers. Four cases are considered in the figures: \( P_{SS} = P_{SD} = 0 \); \( P_{SS} = 0.05 \) and \( P_{SD} = 0.1 \); \( P_{SS} = P_{SD} = 0.1 \); \( P_{SS} = 0.15 \) and \( P_{SD} = 0 \). Please note that the latter three cases satisfy \( 2P_{SS} + P_{SD} = 0.3 \). By (26), it is expected that they will lead to very similar performance.

In Fig. 7 for a given \( N \), we adjust \( \xi \) to achieve \( P_{FA} = 0.001 \) and plot the corresponding \( P_{MD} \). As previously discussed, varying \( \xi \) can only give \( N + 1 \) discrete pairs of \( P_{FA} \) and \( P_D \). Therefore, we obtain \( P_{MD} \) at \( P_{FA} = 0.001 \) by a
piecewise linear interpolation of these pairs. In Fig. 8 we obtain $P_{FA}$ at $P_{MD} = 0.15$ for various $N$ in the same manner.

It can be seen from Fig. 7 that for a constant $P_{FA}$, $P_{MD}$ decreases exponentially with increasing number of cross-check receivers. A similar behavior of $P_{FA}$ for a constant $P_{MD}$ can also be seen in Fig. 8. Besides, both figures clearly demonstrate that $P_{SS}$ deteriorates performance twice as significantly as $P_{SD}$ does. This confirms our theoretical conclusions in the previous section. Additionally, the figures show that even if 15% to 25% of the cross-check receivers are unreliable (a very conservative assumption, with different combinations of $P_{SS}$ and $P_{SD}$), four cross-check receivers suffice to provide as low $P_{FA}$ and $P_{MD}$ as a single reliable cross-check receiver.

V. Experiments

In this section, we conduct field experiments to evaluate authentication performance in real environments. Since Sections III and IV have analyzed and demonstrated the performance of decision aggregation, this section focuses on pair-wise check.

In the experiments, we employ multiple SiGe GN3S samplers and portable antennas to collect raw intermediate frequency (IF) samples of GPS signals. The SiGe front-end is a thumb-sized USB device designed for low-cost software-defined GPS and Galileo receivers. It has a sampling frequency from 4 MHz to 16 MHz and a quantization resolution of 2 bits.
(4 levels). The data are post-processed using our developed software receiver, which is modified from [42]. Snippets of P(Y) codes are extracted from the tracking loops, and then used to compute correlations.

The experiments are conducted in different spatial conditions (urban canyon and open space) and different transport modes (static and moving) with different distance between receivers. In comparison to the experiments in [29] which used static, high-quality front-ends and antennas, our experiments can better evaluate the authentication performance for real applications, especially the GPS receivers in mobile devices and on vehicles.

A. Experiment 1: 3000 kilometers apart, one receiver in urban canyon

The first data set was collected on 27 March 2014. As shown in Fig. 9, one SiGe receiver was in a urban canyon in San Francisco, CA with open sky to the south east. The other receiver was in Urbana, IL with a clear view of the sky. Two receivers were approximately 3000 kilometers apart. Both receivers were static. The San Francisco receiver experienced severe signal blockage and multipath, and was able to track only three satellites with a low SNR. Fortunately, the Urbana receiver was able to track the three satellites so the pair-wise check was possible.

We performed cross-correlation of the P(Y) snippets generated from the data set. Each snippet is 0.5 second long. At a sampling frequency of 4.092 MHz, a snippet contains \( T = 2.046 \times 10^6 \) samples. The snippets are normalized, i.e., the snippets have a zero mean and are scaled such that \( \sigma_a^2 = \sigma_b^2 = 1 \). The correlation shows that \( \Lambda_1 \Lambda_2 \approx 0.00553 \). According to (12) and (13), we chose the threshold \( \zeta = 0.00553/2 \approx 0.00276 \) so that we have the same probability of false alarm and missed detection, i.e., \( \alpha = \beta \).

We injected spoof signal into the raw data from San Francisco receiver starting from 10 seconds. The spoofing signal was initially synchronized to the authentic signal so that the receiver could lock on to both authentic and counterfeit C/A code. Then, the counterfeit C/A code phase moved away from the authentic C/A code phase at a rate of 0.5 chip per second. The receiver tracking loop was dragged by the spoofing signal because the spoofing signal was slightly stronger than the authentic signal.

Figure 10 shows the pair-wise check test statistic \( C \), as defined in (5), before and under the spoofing attack. The test statistic \( C \) is above the threshold \( \zeta \) until the attack starts. As soon as the attack starts, \( C \) quickly drops below \( \zeta \). Due to the relatively low SNR, at some epochs \( C \) is very close to the threshold, with the potential to cause false alarms or missed detection if the threshold was not properly chosen.

This experiment shows that it is possible to use a receiver in urban canyon environments for cooperative authentication, as long as the receivers is able to track at least one satellite. However, the performance deterioration due to a low SNR should be compensated by using more cross-check receivers, as discussed in Sections III and IV.

B. Experiment 2: 22 kilometers apart, one moving receiver

The second data set was collected on 3 April 2014. One SiGe receiver was on a car moving at roughly 45 miles per hour in Rantoul, IL. The other receiver was in Urbana, IL. Two receivers were approximately 22 kilometers apart. Both receivers had a clear view of the sky. Ten satellites were visible to each receivers, and 8 of them were tracked by both receivers.

We performed similar cross-correlation as done in Experiment 1. Because the data was collected at a different sampling frequency, 5.456 MHz, a 0.5-second snippet contains \( T = 2.728 \times 10^6 \) samples. The snippets are normalized. The correlation shows that the estimate of \( \Lambda_1 \Lambda_2 \approx 0.01295 \), and we chose the threshold \( \zeta = 0.01295/2 \approx 0.00648 \).

We injected spoof signal into the raw data from Rantoul receiver in the same way as Experiment 1. The only difference is that the counterfeit C/A code phase moved away from the authentic C/A code phase at a lower rate, 0.375 chip per second.
different transport modes (static and moving). The experiments show that SNR is the major factor affecting pair-wise check performance. A powerful aspect of these results is that even if the cross-check receivers are low-cost, unreliable, and in challenging environments, a modest number of such receivers will match, if not outperform, a single, high-quality, reliable reference receiver in terms of spoofing detection performance.

REFERENCES

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