Urban GPS Integrity Monitoring Using a Graph-SLAM Framework

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Abstract

Vehicles operating in urban environments are prone to GPS faults caused by multipath and satellite broadcast anomalies in multiple satellite channels. We propose a graph-based Simultaneous Localization and Mapping (SLAM) framework to perform receiver Integrity Monitoring (IM) as well as multiple GPS Fault Detection and Isolation.

Utilizing the GPS pseudoranges, we simultaneously localize not only the receiver but also the GPS satellites. To address multiple GPS faults, we design an M-estimator-based cost function and then perform iterative graph optimization via Levenberg Marquardt algorithm. For each satellite, we independently compute its fault probability by evaluating its empirical cumulative distribution. We theoretically derive the protection levels of the receiver position, by calculating the worst-case failure slope that maximizes the eigenvalues of the dominant GPS faults. For different experimental scenarios, namely, ground vehicle and aerial vehicle, we demonstrate that our SLAM-based IM algorithm performs robust integrity assessment as compared to the conventional IM algorithms.

1 Introduction

Recently, there has been a wide-spread utilization of GNSS-based navigation system for urban applications [1] via ground vehicles [2] and aerial vehicles [3]. Assessing the probability of a GNSS satellite fault is a well-formulated framework for aircraft navigation. However, vehicles operating in unpredictable urban environments face additional challenges [4]. Some of the major challenges, as discussed in [5], include static infrastructure, such as, buildings and thick foliage as well as dynamic obstacles, such as, traffic and pedestrians.

This work focuses on two major GNSS faults experienced by the vehicles, namely, satellite and receiver faults. Satellite faults [6] occur due to the anomalies in different GNSS segments, namely, receiver malfunction in the user segment, clock anomalies in the space segment and satellite broadcast anomalies due to the control segment. Due to the lack of satellite visibility from the ground station, the anomalies that occur in the space and control segments require at least a few hours to be rectified. Some of the most recent real-world incidents reporting satellite faults are listed as follows: non-standard codes were transmitted by a satellite, namely, SV49 [7] in 2017 and wrong timing offsets were broadcast by multiple satellites [8] in 2016. The detailed documentation regarding various single/multiple satellite faults that have occurred in the past decade are listed in the literature [9]. In addition, the increasing number of GNSS satellites due to multiple constellations, such as GPS, GLONASS, Galileo and BeiDou increases the probability of multiple faults [11].

Receiver faults, which occur in the presence of dense tall buildings in the urban areas, are caused by the satellite blockage, poor satellite geometry and multipath effects in multiple satellite signals. Recently, a drone inspecting the Millenium Tower in San Francisco dropped from the sky due to the loss of GPS signal [10]. Unlike the satellite faults, the receiver faults are more frequent and therefore, require constant monitoring.

Traditional Receiver Autonomous Integrity Monitoring (RAIM) algorithms, such as, Range Comparison Method (RCM)[12] and Multiple Hypothesis Solution Separation (MHSS) [13] utilize additional ranging information from redundant (> 4) satellite signals to assess the receiver integrity. Considering a prior hypothesis of having multiple faulty measurements simultaneously, MHSS RAJM evaluates the separation between the position estimated by considering all satellites in view and the position computed by excluding the hypothesized faulty satellites. However, with the increase in number of satellites, the implementation of MHSS RAJM becomes quite computationally expensive. While these traditional methods show promising results for applications in civil aviation, they are not directly applicable for vehicles that navigate in urban areas with limited number of non-blocked satellites, poor satellite geometry and the presence of uncorrelated multipath in multiple satellite channels.
Among the existing research on receiver Integrity Monitoring (IM), [14] proposed a RAIM algorithm that calculates the protection levels by estimating the extended Kalman Filter-based position error bound. Even though this approach presented an innovative methodology to calculate the integrity-based protection levels for Kalman Filter framework, there exists some drawbacks that are listed as follows: the algorithm addresses single satellite faults and the fault profiles are only considered to be either step or ramp in nature. Other works, such as, [15]-[17] also utilized the Kalman Filter framework to execute snapshot-based integrity monitoring in urban environments. However, the Kalman Filter framework that propagates based on the jointly distributed nature of the sequential measurements is not an valid assumption in the presence of GNSS faults.

In [18], the authors addressed an interesting perspective that, given the restricted satellite geometry of urban areas, the influence of excluding a faulty satellite might be greater than the fault itself. The authors developed an urban-adapted RAIM algorithm based on the exclusion accuracy filter to account for both poor satellite geometry as well as multipath effects. Prior research [19] utilized a Chi-squared RAIM approach to detect and isolate faults, whose improved performance is later compared with the traditional MHSS. Authors of [20] developed an Isotropy-based Protection Level (IBPL) algorithm to compute the horizontal protection levels based on the range measurement residuals that are modeled to follow an isotropic distribution. But the disadvantage with this algorithm is that the calculated protection levels are highly dependent on the number of available measurements.

The authors of [21] adapt the traditional RAIM techniques to the urban context by assessing the position, speed and map-matching-based integrity. This algorithm utilizes additional resources, such as, roadmap database and wheel speed sensor to perform fault detection, but these checks are executed on higher-level metrics, such as, position and speed that limit the accuracy of the computed protection levels. Another work [22] proposed an integrity approach based on the Random Sample Consensus (RANSAC) algorithm that is capable of detecting multiple satellite failures. However, the major disadvantage of this subset evaluation-based algorithm is related to the high computational cost requirements and the difficulties involved in its practical implementation. In addition, paper [23] provides a detailed literature review regarding the various existing approaches to address integrity assessment in urban areas.

1.1 Our main contributions

We propose a Simultaneous Localization and Mapping (SLAM)-based IM algorithm that performs multiple Fault Detection and Isolation (FDI) as well as integrity assessment of vehicles operating in the urban areas. We describe our SLAM-based IM algorithm by considering only one GNSS constellation, i.e., GPS, but the same framework can be applied to multiple GNSS constellations. In our prior work [24], we designed a SLAM [3] framework to simultaneously estimate the Position, Velocity and Time (PVT) of the receiver and the GPS satellites, which are later utilized to perform multiple FDI by individually evaluating the measurement residuals. In this work, we extend our algorithm to not only perform multiple FDI but also compute the protection levels associated with the estimated receiver position. In particular, the contributions of our proposed SLAM-based IM algorithm are listed as follows:

1. We design a graph-SLAM framework that simultaneously estimates the PVT of not only the receiver but also the GPS satellites, thereby, accounting for the multiple GPS faults caused by both multipath and satellite broadcast anomalies.

2. To account for the error distributions induced by multiple GPS faults, we perform graph optimization by formulating the cost function as a combination of the Huber M-estimator [25], which guarantees global convergence and the Tukey bisquare M-estimator [26], which ensures accurate estimation of not only the PVT of the receiver but also the satellites.

3. To address the non-existence of an analytical closed form solution for the non-linear cost function, which is based on an M-estimator formulation, we consider an iterative technique based on Levenberg Marquardt [27] algorithm to execute the graph optimization.

4. We perform the graph optimization in two threads: one is sub-graph optimization, which is executed on the most recent time instants while the other is full-graph optimization, which is executed periodically to bind the overall drift errors in the estimated PVT of the receiver and satellites.

5. For each satellite, we independently compute an on-the-fly empirical Cumulative Distribution Function (CDF), which is used to evaluate the measurement residual-based test statistic, thereby, both detecting and isolating multiple GPS faults.

6. Utilizing the detected GPS faults as a prior hypothesis, we theoretically compute the protection levels of the receiver position by evaluating the worst-case failure slope [28] that maximizes the eigenvalues of the dominant GPS faults, i.e., satellite and receiver faults.
7. We also validate our algorithm by conducting extensive experiments, namely, adding multiple simulated broadcast anomalies to the open-sky data collected using a ground vehicle and flying an aerial vehicle in an urban area prone to multipath. We also demonstrate the tighter protection levels achieved using our proposed algorithm as compared to the traditional RCM [12].

The rest of the paper is organized as follows: Section II describes our SLAM-based IM algorithm and its key characteristics; Section III experimentally validates the performance of our algorithm in detecting and isolating multiple faults as well as assessing the receiver integrity while accurately estimating the receiver location; Section IV concludes the paper.

2 SLAM-based IM Algorithm

In this section, we outline the architecture of our SLAM-based IM algorithm and then explain the algorithmic details.

SLAM [29] is a well-known technique in robotics. It utilizes the sensor measurements to estimate the landmarks in a three-dimensional (3D) map while simultaneously localizing the robot within it. Analogous to this, we design a Graph-SLAM [30] framework, where the robot is our GPS receiver and the GPS satellites are considered as landmarks in the map. We aim to simultaneously update the PVT of the receiver and landmarks in the map, i.e., GPS satellites.

Our algorithm utilizes sequential measurements in an optimization framework to detect as well as mitigate the effect of multiple GPS faults without added computational complexity. In this work, we refer to fault as the condition where the measurement error associated with any satellite and caused by either broadcast anomalies and/or multipath effects is above a particular pre-defined threshold.

In this work, integrity is defined as a measure of trust, which can be placed in the correctness of the information supplied by the navigation system, thereby, indicating the confidence in the estimated PVT of the receiver [23]. One parameter that quantifies the concept of integrity is termed as the Integrity Risk (IR) [31]. IR, given by Eq. (1), is defined as the maximum probability with which a receiver is allowed to provide position failures not detected by the integrity monitoring system.

\[ p_{IR} = p(|\epsilon_0| > l \cap |q| < \nu), \]  

where \( \epsilon_0 \) denotes the estimator error on the state vector, \( l \) denotes the pre-defined Alert Limit (AL), which indicates the error tolerance not to be exceeded without issuing a limit. Similarly, \( q \) denotes the detection test statistic based on measurements and \( \nu \) represents pre-defined detection threshold.

Similarly, another integrity parameter coined as protection level, as described in Eq. (2), represents the statistical error bound computed so as to guarantee that the probability of absolute position error exceeding a certain value is smaller than or equal to the target IR [31]. If the protection levels are greater than the value of AL specified by applications, then the alert triggers. Using our SLAM-based IM algorithm, we focus on estimating the positioning protection levels based on the probability derived from the pre-defined integrity requirements, namely, false alarm and misdetection.

\[ p(|\epsilon_0| > PL|Z|) < p_{IR}, \]  

where \( Z \) denotes the measurement, based on which the protection levels \( PL \), associated with the error vector \( \epsilon_0 \) are computed.

2.1 Our Architecture

We propose our SLAM-based IM architecture, shown in Fig. 1 to assess the positioning protection levels while accurately estimating the PVT of both the receiver and satellites. In addition, we also detect and isolate multiple GPS faults. The details are described as follows:

1. We initialize our 3D graph via the PVT of the receiver and all the visible GPS satellites that are computed using the existing GPS algorithms [32];

2. We obtain measurements from different modules, which include the receiver motion model, satellite navigation message and pseudoranges corresponding to the GPS satellites. These inputs are provided to the graph optimization module;

3. In the graph optimization module, we minimize the total error, which is formulated as a cost function based on M-estimator [33], using the Levenberg-Marquardt [27] algorithm. The total error comprises of terms that represent the residuals in the GPS measurements, receiver and satellite dynamics weighted by the corresponding fault probability computed at the previous time instant;
4. For each satellite, we individually compute the fault probability by evaluating the test statistic against the empirical cumulative distribution computed on-the-fly;

5. Based on the GPS faults detected, we evaluate the protection levels associated with the receiver position by calculating the worst-case failure slope that maximizes the eigenvalues that correspond to the most dominant GPS faults, i.e., either satellite or receiver faults;

6. Later, based on the fault probability associated with each satellite, we repeat the above steps to iteratively build the graph by simultaneously localizing the PVT of receiver and satellites.

![Architecture of our SLAM-based IM algorithm.](image)

**Figure 1:** Architecture of our SLAM-based IM algorithm.

### 2.2 Input Measurements

The measurements provided as input to our SLAM-based IM algorithm are explained as follows

1) **GPS signal model**

Considering the GPS satellite signals received from \( N \) satellites in view of the receiver and processing them via existing algorithms [32] to individually estimate the satellite signal parameters, we compute the pseudoranges, denoted by \( \rho^i \), as

\[
\rho^i = \| \mathbf{p}^i - \mathbf{p} \| + (c\delta t - c\delta t^i) + c\delta t_{atmos}^i + b^i + \eta^i,
\]

where

- \( \mathbf{p} \) and \( \mathbf{p}^i \) denotes the 3D position of the receiver and the \( i^{th} \) satellite, respectively;
- \( c\delta t \) and \( c\delta t^i \) represents the receiver clock bias and \( i^{th} \) satellite clock corrections, respectively;
- \( c \) denotes the speed of light;
- \( c\delta t_{atmos}^i \) denote the atmospheric errors caused due to ionospheric and tropospheric effects;
- \( b^i \) indicates the GPS receiver faults associated with the pseudorange measurement of the \( i^{th} \) satellite.
- \( \eta^i \) represents the measurement noise associated with pseudorange of the \( i^{th} \) satellite;
Based on Eq. (3), we define the GPS measurement model that computes the expected pseudorange as

\[ h(X_t, Y^i_t) = \|p^i - p\| + (c\delta t - c\delta t^i) \]  

(4)

where

- \( X_t \) denotes the \( 8 \times 1 \) receiver state vector at \( t^{th} \) time instant comprising of 3D position, 3D velocity, clock bias and clock drift, such that \( X_t = [p, c\delta t, \dot{p}, c\dot{\delta} t] \);
- \( Y^i_t \) denotes the \( 8 \times 1 \) state vector of the \( i^{th} \) satellite comprising of its 3D position, 3D velocity, clock bias and clock drift corrections, such that \( Y^i_t = [p^i, c\delta t^i, \dot{p}^i, c\dot{\delta} t^i] \), \( i \in \{1, \cdots, N\} \);
- \( \dot{p} \) and \( \dot{p}^i \) denotes the velocity of the receiver and \( i^{th} \) satellite, respectively, such that, \( \dot{p} = [\dot{x}_r, \dot{y}_r, \dot{z}_r] \) and \( \dot{p}^i = [\dot{x}^i, \dot{y}^i, \dot{z}^i] \);
- \( c\delta t \) and \( c\delta t^i \) denotes the receiver clock drift and \( i^{th} \) satellite clock drift corrections respectively;

2) Receiver and satellite motion models

In addition to pseudoranges, we provide the motion model of the receiver and navigation message from the satellite orbital model as inputs to our SLAM-based IM algorithm. In particular, the control input obtained from the receiver motion model depends on the dynamics of the vehicle platform, such as constant velocity, zero velocity, constant acceleration, and so on. The receiver motion model, denoted by \( g(.) \), is given by

\[ \hat{X}_t = g(u_{R,t}, X_{t-1}, Q_t), \]  

(5)

where

- \( g(.) \) represents the receiver motion model that is set based on the vehicle dynamics. The corresponding implementation details are provided later in the Section 3.1;
- \( \hat{X}_t \) denotes the predicted receiver state vector at \( t^{th} \) time;
- \( u_{R,t} \) denotes the control input of the vehicle on which the receiver is mounted;
- \( Q_t \) denotes the dynamic process noise covariance matrix that accounts for the vehicle accelerations.

Similarly, the satellite orbital model, denoted by \( f(.) \), estimates the predicted state vector of each \( i^{th} \) satellite, i.e., \( \hat{Y}^i_t, i \in \{1, \cdots, N\} \). This is computed using the necessary satellite signal information decoded from the broadcast navigation message [12] and is represented by \( u^i_t \).

![Figure 2: Framework of the Graph-SLAM module. The three measurement nodes, indicated in orange, correspond to the receiver control input \( u_{R,t} \), satellite broadcast message \( u^i_t \) and pseudoranges \( \rho^i_t \). The variable nodes to be estimated, indicated in blue, represent the sequential time series of the receiver \( X_{1:t} \), satellites \( Y^{1:N}_{1:t} \) and fault probabilities \( I^{1:N}_{0:t-1} \).](image-url)
2.3 Graph formulation

Our graphical framework, seen in Fig. 2, consists of two types of units, namely, variable nodes, indicated in blue and measurement nodes, indicated in orange. The orange nodes represent the measurements that include GPS pseudoranges, receiver motion model and satellite navigation message. The functionality of various nodes are described as follows:

1. The blue nodes in the receiver layer depict the sequential time series of the receiver’s PVT $X_{1:t}$.
2. Similarly, each sub-layer in the satellite layer consists of blue nodes, which represent the sequential PVT time series of the satellites $Y_{i,1:t}$, $i \in \{1, \ldots, N\}$.
3. Both the receiver and satellite layers are collectively constrained via GPS pseudoranges depicted by $\rho_i^t$.
4. At any time instant, the receiver node is constrained by its control input $u_{R,t}$ and similarly, the satellite nodes are constrained by their broadcast message $u_i^t$.
5. In addition, each satellite node $Y_{i,t}$ is connected to a corresponding fault node $I_{i,t-1}$, whose values range between $0 - 1$, where 0 indicates low and 1 indicates high fault probability associated with the $i^{th}$ satellite.

In this work, the symbol $\hat{\cdot}$ associated with a variable, such as the state vector of receiver and satellites, represents its predicted state whereas the symbol $\bar{\cdot}$ associated with a variable denotes the estimated state via our SLAM-based IM algorithm.

2.4 Graph optimization module

We execute optimization in two threads: one is sub-graph optimization and the other is full-graph optimization. In sub-graph optimization, at each instant, we optimize the cost function on the recent time instants to obtain the corresponding PVT of both the receiver and satellites. In full-graph optimization, we perform global graph optimization by minimizing the cost function over all the time instants.

1) M-estimator-based cost function

Various proposed SLAM algorithms [34] in robotics rely on minimizing the least-squared residual-based cost function by modeling these errors as a Gaussian distribution. However, during GPS faults, the errors in outlier measurements exhibit large tails that no longer follow a Gaussian distribution. Therefore, we utilize a combination of the Huber M-estimator [25] and the robust redescending M-estimator known as the Tukey bisquare M-estimator [26] to formulate the cost function.

The bisquare M-estimator

$$\Lambda_B(v, \omega) = \begin{cases} (\omega^{-1}v)^2 \left( 1 - (\frac{(\omega^{-1}v)^2}{k_B^2})^3 \right) & |v| \leq k_B \vspace{5pt} \\
0 & \text{otherwise} \end{cases},$$

where $k_B$ denotes the bisquare constant. In addition, $v$ denotes the 2-norm residual and $\omega$ denotes the covariance associated with the residual.

In contrast, the Huber M-estimator $\Lambda_H(v, \omega)$ exhibits global convergence as seen in Eq. (7). However, they have a lower accuracy than the biquare estimators because they assign equal weights to all the measurement residuals.

$$\Lambda_H(v, \omega) = \begin{cases} 0.5 (\omega^{-1}v)^2 & |v| \leq k_H \\
k_H \left( (\omega^{-1}v) - k_H \right) & \text{otherwise} \end{cases}.$$

where $k_H$ denotes the Huber constant.

To account for the above-mentioned limitations, we switch between the two estimators based on different operating conditions. During initialization and when the number of satellites $N \leq 4$, we utilize the Huber M-estimator $\Lambda = \Lambda_H$ to ensure convergence; in all the other conditions, we opt for the bisquare M-estimator $\Lambda = \Lambda_B$ to achieve better accuracy.

2) Sub-graph optimization thread

In the sub-graph optimization thread, we optimize the graph at each time instant, by formulating an error-based cost function
using Huber and bisquare M-estimators, which are described earlier in the Section 2.4.1. Our M-estimator-based cost function, denoted by \( e_t \), consists of three components [35] and is given by

\[
e_t(\theta_t) = e_t(X_t, Y^1_t, \ldots, Y^N_t)
\]

\[
= \sum_{i=1}^{N} \Lambda \left( \left( \rho^i_t - h(X_t, Y^i_t) \right), I_{i-1} + \sigma^i_t \right) + \Lambda \left( \| X_t - g(u_{R,t}, \bar{X}_{t-1}, Q) \|, \hat{\Sigma}_t \right)
\]

\[
+ \sum_{i=1}^{N} \Lambda \left( \| Y^i_t - f(u^i_t) \|, I_{i-1} \right),
\]

\[
\bar{\theta}_{t-T:t} = \left[ \bar{X}_{t-T:t}, \bar{Y}^1_{t-T:t}, \cdots, \bar{Y}^N_{t-T:t} \right]
\]

\[
= \arg \min_{\theta_{t-T:t}} \left( \sum_{s=t}^{t} e_s(\theta_s) \right),
\]

where

- \( \theta_t \) denotes the overall state vector representing the PVT of the receiver and all satellites, given by \( \theta_t = [X_t, Y^1_t, \cdots, Y^N_t] \) and is estimated during the graph optimization. Given \( N \) number of satellites, \( \theta_t \) is a \( S(N+1) \times 1 \) state vector;

- \( \hat{\Sigma}_t \) denotes the predicted covariance matrix of the receiver state vector at the \( t^{th} \) time instant;

- \( \sigma^i_t \) denote the covariances of the measured pseudorange associated with the \( i^{th} \) satellite;

- \( T \) denotes the number of time instants utilized in the sub-graph optimization thread;

- \( \bar{\theta}_t \) denotes our SLAM-based IM estimate of the overall state vector computed during the sub-graph optimization.

In Eq. (9), using graph optimization, we estimate the unknown PVT of the receiver \( X_t \) and satellites \( Y^1_t:Y^N_t \), which are collectively represented by the overall state vector \( \theta_t = [X_t, Y^1_t, \cdots, Y^N_t] \). The first term in the loss function \( e_t \), given in Eq. (8), denotes the summation of residual error between the pseudorange measurement \( \rho^i_t \) and that computed from the GPS measurement model \( h(\cdot, \cdot) \) across the satellites in view, as described in Eq. (3). In the second term, we compare the receiver state vector to be optimized with that of the receiver motion model \( g(\cdot) \), as described in Eq. (5). In the third term, we compare the satellite position to be optimized with that of the satellite orbital dynamics model \( f(\cdot) \) decoded from the broadcast navigation message, which is described earlier in Section 2.2.2. Thereafter, we carry out optimization on the most recent \( T \) measurements to compute the SLAM-based IM estimate of the overall state vector \( \bar{\theta}_{t-T:t} = [X_{t-T:t}, Y^1_{t-T:t}, \cdots, Y^N_{t-T:t}] \). The details regarding the iterative steps executed to perform the above graph optimization are explained later in the Section 2.5.

3) Full-graph optimization thread

The full-graph optimization is performed to bind the overall drift errors and correct the PVT of receiver and satellites based on the global map generated. This occurs at a slower update rate as compared to sub-graph optimization so as to reduce the overall computational complexity. We execute mapping thread to constrain the drifts and obtain the globally corrected estimates of the receiver \( \bar{X}_{1:t} \) and satellites \( \bar{Y}^1_{1:t} \) given by

\[
\bar{\theta}_{1:t} = [\bar{X}_{1:t}, \bar{Y}^1_{1:t}, \cdots, \bar{Y}^N_{1:t}],
\]

\[
= \arg \min_{\theta_{1:t}} \left( \sum_{s=1}^{t} e_s(\theta_s) \right).
\]

2.5 Iterative graph optimization and covariance estimation

Since Eq. (8) is non-linear function of \( \theta_t \), an analytical solution \( \bar{\theta}_t \) for the minimum of Eq. (9) does not exist. Therefore, we opt for an iterative technique to estimate \( \theta_t \) with a pre-determined convergence condition as the stopping criterion. We describe the equations for the \( t^{th} \) time instant, which can be extended to \( T \) or \( t \) time instants based on the initialization conditions of the sub-graph and full-graph optimization threads, respectively.
By considering a forward model as described in [36], the non-linear function $e_t(\theta_n)$ is iteratively optimized using an Levenberg Marquardt minimizer, at each instant, starting with $n = 0$ as follows:

$$
\theta_{n+1,t} = \theta_{n,t} + \left( H_{n,t}^T S_t^{-1} H_{n,t} + R_t + \beta_{n,t} \text{diag}(H_{n,t}^T H_{n,t}) \right)^{-1} \left[ H_{n,t}^T S_t^{-1} (Z_t - h(\theta_{n,t})) + R_t (\hat{\theta}_t - \theta_{n,t}) \right],
$$

(11)

with

$$
K_{n,t} = \left[ \begin{array}{cccc}
0_{8 \times 8} & 0_{8 \times 8} & \cdots & 0_{8 \times 8} \\
0_{8 \times 8} & w(\hat{X}_t, I_t) I_8 & \cdots & 0_{8 \times 8} \\
\vdots & \vdots & \ddots & \vdots \\
0_{8 \times 8} & 0_{8 \times 8} & \cdots & w(\hat{Y}_t^N, I_t^N) I_8
\end{array} \right],
$$

and

$$
Z_t = \left[ \begin{array}{cccc}
h(X, Y^i) & \vdots & \vdots & \vdots \\
\vdots & \rho^i - c \delta t_{atmos} & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
h(X, Y^N) & \vdots & \vdots & \vdots \\
\vdots & \rho^N - c \delta t_{atmos} & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
\end{array} \right],
$$

(12)

where

- $h(\theta)$ denotes the concatenated vector of the GPS measurement models $h(X, Y^i) \forall i \in \{1, \cdots, N\}$ defined in the Eq. (3), evaluated at $\theta = \theta_{n,t}$;

- $H_n$ denotes the linearized matrix of $h(\theta)$ with respect to $\theta$ and is evaluated at $\theta = \theta_{n,t}$;

- $Z_t$ denotes the measurement vector stacked with the corrected pseudoranges obtained from $N$ satellites. The atmospheric effects are addressed using the existing models [37], such as Klobuchar model;

- $\beta_{n,t}$ denotes the iterative damping factor associated with the Levenberg Marquardt algorithm;

- $w(\upsilon, \omega)$ denotes the M-estimator-based weight function, such that, $w = \frac{1}{\upsilon} \frac{\partial \Lambda}{\partial \upsilon}$, where $\Lambda$ is obtained from Eqs. (6) and (7);

- $S_t^{-1}$ represents the M-estimator-based weight function for the GPS pseudoranges $Z_t$, such that, $S_t^{-1} = \text{diag}\{ w(\rho^i - c \delta t_{atmos}, \sigma^i) \forall i \in \{1, \cdots, N\} \}$;

- $0_{8 \times 8}$ and $I_8$ indicates the zero and identity matrices, respectively, both of size $8 \times 8$;

- $R_t$ represents the M-estimator-based weight function for the predicted overall state vector $\hat{\theta}_t$, which is obtained from the receiver and satellite motion models;

- $V_{n,t}$ denotes the pseudo-inverse matrix, given by $V_{n,t} = \left( H_{n,t}^T S_t^{-1} H_{n,t} + R_t + \beta_{n,t} \text{diag}(H_{n,t}^T H_{n,t}) \right)^{-1}$;

- $K_{n,t}$ denotes the gain matrix, given by $K_{n,t} = V_{n,t} H_{n,t}^T S_t^{-1}$.

During $n^{th}$ iteration, for a damping factor $\beta_{n,t} = \beta_0$, if $e_t(\theta_{n+1,t}) > e_t(\theta_{n,t})$, then we discard the estimate $\theta_{n+1,t}$ and repeat the $n^{th}$ iteration with a larger damping factor, i.e., $\beta_{n,t} > \beta_0$. Otherwise, we accept $\theta_{n+1,t}$ as an updated estimate and the corresponding lower value of the damping factor is assigned for the subsequent $(n+1)^{th}$ iteration, such that, $\beta_{n+1,t} = \beta_{n,t}$. Once the pre-defined convergence condition for the $t^{th}$ time instant is attained, say after $W_t$ iterations, the value of $\theta_{W_t,t}$ is assigned as our SLAM-based IM estimate of the PVT of the receiver and satellites at the time instant $t$, described in Eq. (9), such that, $\hat{\theta}_t = \theta_{W_t,t}$. The convergence condition is set during the initialization stage, which is discussed later in the Section 3.1.
Similar to the overall state vector \( \hat{\theta}_t \) that is estimated via iterative graph optimization, as seen in Eq. (11), we also iteratively estimate its associated covariance matrix. To perform this, we consider a transformation matrix that propagates the covariances associated with the overall measurement vector \( [Z, \theta]^T \) to the PVT domain. Therefore, the transformation matrix, represented by \( J_t \), is expressed as follows:

\[
[\Sigma_t \sim \Omega_t] = J_t \begin{bmatrix}
\sigma^1 & \ldots & 0 & 0_{1 \times 8} & 0_{1 \times 8N} \\
\vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \ldots & \sigma^N & 0_{1 \times 8} & 0_{1 \times 8N} \\
0_{8 \times 1} & \ldots & 0_{8 \times 1} & \Sigma_t & 0_{8 \times 8N} \\
0_{8N \times 1} & \ldots & 0_{8N \times 1} & 0_{8N \times 8} & \Omega_t
\end{bmatrix} J_t^T,
\]

(13)

where

- \( \Sigma_t \) denotes the iterative graph optimized estimate of the covariance of the receiver state vector \( \hat{X}_t \);
- \( \Omega_t \) denotes the predicted and estimated covariances of the satellite state vectors \( \hat{Y}_i \) \( \forall i \in \{1, \ldots, N\} \);
- \( J_t \) denotes the \( 8(N+1) \times (N+8(N+1)) \) transformation matrix, such that any \( i^{th} \) row and \( k^{th} \) column entry of \( J_t \), i.e., \( J_{jk} = \frac{\partial x_j}{\partial y_k}, \forall x \in \theta, y \in \left[ Z, \hat{\theta} \right]^T \).

Therefore, we iteratively estimate the transformation matrix \( J_t \) by calculating the derivative of Eq. (11), with respect to the total measurement vector \( [Z, \theta]^T \) as follows:

\[
J_{n+1,t} = \begin{cases} 
0, & \text{if } n+1 = 0 \\
J_{n,t} + K_{n,t}(I - H_{n,t}P_{n,t}) + M_{n,t}R_{t}(I - J_{n,t}), & \text{otherwise}
\end{cases}
\]

(14)

At \( t^{th} \) time instant, by iterating over the Eq. (14) for the same number of steps, i.e., \( W_t \) as that required to achieve convergence in case of overall state vector \( \hat{\theta}_{n,t} \), we compute our SLAM-based IM estimate of the transformation matrix, defined by \( \hat{J}_t \), such that \( \hat{J}_t = J_{W_t,t} \). Thereafter, by substituting \( \hat{J}_t \) in the Eq. (13), we evaluate the covariance matrices associated with our SLAM-based IM estimates of the state vector of the receiver and satellites, represented by \( \Sigma_t \) and \( \Omega_t \), respectively.

### 2.6 Fault Detection and Isolation

Next, to detect and isolate multiple faults, we individually compute the test statistic \( \gamma^i \) for each satellite, as seen in Eq. (15), that comprises of two terms: one term is the difference between expected pseudorange computed using our SLAM-based IM estimate of the receiver state vector \( \hat{\theta}_t \), and the corresponding measured pseudorange. The other term indicates the difference between PVT of satellite \( Y_i \) estimated using the sub-graph optimization thread and that obtained using the broadcast ephemeris decoded from the navigation message.

\[
\gamma^i = ||\bar{Y}_i - f(u^i)|| + ||Z_i - h(X_t, \bar{Y}_i)||
\]

(15)

In the absence of GPS faults, the test statistic \( \gamma^i \) lies on a Gaussian distribution \( \Gamma^i \sim N(,.) \), which is justified based on the GPS signal model structure [12]. Note that, our multiple FDI algorithm operates on an assumption of Gaussian error distribution in the case of fault-free. During fault-free conditions, we consider the most recent test statistic values and perform an on-the-fly empirical estimation of the CDF, denoted by \( \Phi^i_{t-1,t} \) for each \( t^{th} \) satellite independently. Based on [38], at \( t^{th} \) time instant, by utilizing the empirical CDF of each \( i^{th} \) GPS satellite, we evaluate its fault probability \( I_i^t \) as follows:

\[
I_i^t = 4 \left( \Phi^i_{t-1,t}(\gamma^i) - 0.5 \right)^2,
\]

(16)

where

- \( \Phi^i_{t-1,t} \) denotes the empirical CDF computed during fault-free conditions;
- \( \gamma^i \) denotes the test statistic that is being evaluated against the estimated \( \Phi^i_{t-1,t} \) to detect the presence/absence of GPS faults associated with the \( i^{th} \) satellite. 


2.7 Receiver Integrity Monitoring

To compute the protection levels associated with the estimated position of the receiver, firstly, we re-write the measurement vector defined in Eqs. (3) and (5) as follows

\[ Z = \begin{bmatrix} \hat{H} \\ I_{(8N+5)} \end{bmatrix} \theta + \begin{bmatrix} b_{gps} \\ b_{mm} \end{bmatrix} + \begin{bmatrix} \eta_{gps} \\ \eta_{mm} \end{bmatrix}, \]  

(17)

where

- \( \hat{H} \) denotes the linearized matrix of \( h(\theta) \) with respect to \( \theta \) and is evaluated at \( \theta = \bar{\theta}_t \);
- \( b_{gps} \) denotes the fault vector associated with the GPS pseudoranges, such that, \( b_{gps} = [b^1, \ldots, b^N] \), where \( b^i \) indicates the receiver fault in the pseudorange measurement of the \( i^{th} \) satellite and is defined in Eq. (3);
- \( b_{mm} \) represents the fault vector associated with the receiver and satellite motion model, which are described in the Section 2.2.2;
- \( \eta_{gps} \) denotes the noise vector given by \( \eta_{gps} = [\eta^1, \ldots, \eta^N] \), where \( \eta^i \), defined in Eq. (3), denotes the noise in pseudorange measurement of the \( i^{th} \) satellite and similarly \( \eta_{mm} \) denotes the noise vector associated with the receiver and satellite motion models.

According to [28], in the presence of multiple simultaneous faults, the worst-case fault vector must be determined, such that, it maximizes the integrity risk bound for a given fault hypothesis. A fault vector is characterized by three components: hypothesis, magnitude and direction, of which the fault hypothesis and magnitude are discussed earlier in Section 2.6. In this subsection, we determine the worst-case fault direction, which in-turn maximizes the failure slope for a given integrity requirement, i.e., probability of false alarm and misdetection.

The square of failure mode slope, denoted by \( m^2 \), is defined as the ratio of the squared mean of the estimate error in positioning over the squared mean of the measurement residual. In our analysis, we assume the test statistic \( q \), which is considered as a squared 2-norm of the measurement residual \( r \), to follow a centered chi-squared distribution in case of fault-free cases and a non-centered chi-squared distribution in the presence of faults [28],[39],[40].

\[ q = r^T r = \begin{cases} \chi^2_N, & \text{if fault-free} \\ \chi^2_{(N,\lambda^2)}, & \text{if faulty} \end{cases}, \]  

(18)

where \( \lambda \) denotes the non-centrality parameter associated with the non-centered chi-squared distribution representing the GPS faults caused either due to multipath or broadcast anomalies.

Firstly, we re-write the estimate \( \bar{\theta} \) derived in Eq. (11) obtained by minimizing the error \( e_t(\bar{\theta}_t) \) in Eq. (8) as follows:

\[ \bar{\theta}_t = K Z_t + V R \bar{\theta}_t, \]

\[ = \bar{\theta}_{gps,t} + \bar{\theta}_{mm,t}, \]  

(19)

where

- \( \bar{\theta}_{gps,t} \) denotes the component of the SLAM-based IM estimated error in the overall state vector \( \bar{\theta}_t \) caused by the error in GPS pseudoranges, such that, \( \bar{\theta}_{gps,t} = K_t Z_t \) where \( V = (\hat{H}^T S_t^{-1} \hat{H} + R_t + \beta \text{diag}(\hat{H}^T \hat{H}))^{-1} \), \( K = V \hat{H} S_t^{-1} \) and \( R_t \) as described in Eq. (12);
- \( \bar{\theta}_{mm,t} \) denotes the component of the SLAM-based IM estimated error in the overall state vector \( \bar{\theta}_t \) caused by the error in motion models, such that, \( \bar{\theta}_{mm,t} = VR_t \bar{\theta}_t \).

For simplicity, we drop the subscript \( t \) for the subsequent analysis. We define a vector \( \alpha \) to extract the receiver 3D position from the overall state vector \( \theta \), such that \( \alpha = [1_{3 \times 1}, 0_{(8N+5) \times 1}] \). Thereafter, from Eqs. (17) and (19), we derive the estimate
error in positioning $\epsilon$ as

$$
\epsilon = \alpha^T (\tilde{\theta} - \theta) = \left( \alpha^T K \begin{bmatrix} Z & \theta \end{bmatrix} - \alpha^T \theta \right),
$$

$$
= \alpha^T \left( KH + VR - I \right) \theta + \alpha^T K \left( b + \eta \right),
$$

$$
\approx \alpha^T K (b_{gps} + \eta_{gps}) + \alpha^T VR (b_{mm} + \eta_{mm}),
$$

where we approximate $KH \approx I$ and $VR\theta \approx 0$. Therefore, the total estimate error in positioning $\epsilon$ is represented as two components, such that,

- $\epsilon_{gps}$ denotes the component of the total estimate error in positioning $\epsilon$ caused by the GPS pseudoranges, such that, $\epsilon_{gps} = \alpha^T K (b_{gps} + \eta_{gps})$;
- $\epsilon_{mm}$ denotes the component of the total estimate error in positioning $\epsilon$ caused by the receiver and satellite motion models, such that, $\epsilon_{mm} = \alpha^T VR (b_{mm} + \eta_{mm})$.

Next, we derive the measurement error as

$$
r = \begin{bmatrix} Z \\ \tilde{\theta} \end{bmatrix} - \begin{bmatrix} H \\ I_{(N+8)} \end{bmatrix} \theta,
$$

$$
= \begin{bmatrix} Z \\ \tilde{\theta} \end{bmatrix} - \begin{bmatrix} H \\ I_{(N+8)} \end{bmatrix} \begin{bmatrix} K & VR \end{bmatrix} \begin{bmatrix} Z \\ \theta \end{bmatrix},
$$

$$
\approx \begin{bmatrix} (I - \tilde{H}K)Z + \tilde{H}VR\tilde{\theta} \\ KZ + (I - VR)\tilde{\theta} \end{bmatrix} = \begin{bmatrix} r_{gps} + r_{g-m} \\ r_{m-g} + r_{mm} \end{bmatrix},
$$

where the total measurement residual $r$ can be represented in terms of four components, such that,

- $r_{gps}$ and $r_{g-m}$ denotes the components of the pseudorange residual caused by the estimate error component $\epsilon_{gps}$ and $\epsilon_{mm}$, respectively, and are given by $r_{gps} = \left( I - \tilde{H}K \right)Z$ and $r_{g-m} = \tilde{H}VR\tilde{\theta}$;
- $r_{mm}$ and $r_{m-g}$ denotes the components of the motion model residual caused by the estimate error component $\epsilon_{gps}$ and $\epsilon_{mm}$, respectively, and are given by $r_{m-g} = KZ$ and $r_{mm} = \left( I - VR \right)\tilde{\theta}$.

Based on the Eqs. (20) and (21), the total failure mode slope $m^2$ is given by

$$
m^2 = \frac{\epsilon^T \epsilon}{r^T r},
$$

where

- $\epsilon^T \epsilon$ is obtained from Eq. (20), such that, $\epsilon^T \epsilon = (\epsilon_{gps} + \epsilon_{mm})^T (\epsilon_{gps} + \epsilon_{mm}) = \epsilon_{gps}^T \epsilon_{gps} + 2\epsilon_{gps}^T \epsilon_{mm} + \epsilon_{mm}^T \epsilon_{mm}$;
- $r^T r$ is obtained from Eq. (21), such that, $r^T r = (r_{gps} + r_{g-m})^T (r_{gps} + r_{g-m}) + (r_{m-g} + r_{mm})^T (r_{m-g} + r_{mm})$.

For a vehicle operating in urban areas, the broadcast anomalies are much less frequent and are of significantly higher magnitude than the multipath effects. Therefore, it is justified to first compare the residual errors in pseudoranges with respect to the residual errors in motion model estimates to choose the more dominant GPS faults between satellite and receiver faults. If the pseudorange residuals are more dominant, then worst-case failure slope is calculated under the assumption of multipath effects and otherwise, worst-case failure slope is calculated under the assumption of broadcast anomalies. The details regarding both these cases are explained below.
1) Worst-case failure slope under multipath effects

During the presence of multipath effects in urban areas, the behavior of measurement error \( r \) is dominated by the error in pseudoranges \( Z \) as compared to the motion model \( \hat{\theta} \). Therefore, we consider \( \epsilon_{gps} \gg \epsilon_{mm}, r_{gps} \gg r_{g-m} \) and \( r_{m-g} \gg r_{mm} \), based on which the failure mode slope \( m^2 \) described in Eq. (22) is written as

\[
m^2 = \frac{\epsilon_{gps}^T \epsilon_{gps} + 2 \epsilon_{gps}^T \epsilon_{mm} + m_{gps}^2 + 2 m_{m-g}^2 \kappa_1}{1 + \kappa_1},
\]

where

- \( \kappa_1 \) denotes the ratio between the measurement residuals, namely, \( r_{gps} \) and \( r_{m-g} \), and given by \( \kappa_1 = \frac{r_{gps}^T r_{gps}}{r_{m-g}^T r_{m-g}} \);
- \( m_{gps}^2 \) denotes the failure slope component obtained by considering the estimate error in the overall state vector caused by GPS pseudoranges, i.e., \( \epsilon_{gps} \) and GPS component of the pseudorange residual, i.e., \( r_{gps} \). This is given by
  \[
m_{gps}^2 = \frac{\epsilon_{gps}^T \epsilon_{gps}}{r_{gps}^T r_{gps}},
\]
- \( m_{m-g}^2 \) denotes the failure slope component obtained by considering the estimate error in the overall state vector, i.e., both \( \epsilon_{gps} \) and \( \epsilon_{mm} \) and GPS component of the motion model-based measurement residual, i.e., \( r_{m-g} \). This is given by
  \[
m_{m-g}^2 = \frac{\epsilon_{gps}^T \epsilon_{mm}}{r_{m-g}^T r_{m-g}}.
\]

Given the range of \( \kappa_1 \) is from \([0, \infty)\), during the presence of multipath, our SLAM-based IM estimate of the worst-case total failure mode slope \( \bar{m}^2 \), described in Eq. (22), is given by

\[
\bar{m}^2 = \max(\bar{m}_{gps}^2, 2 \bar{m}_{mm}^2)
\]

where

- \( \bar{m}_{gps}^2 \) denotes our SLAM-based IM estimate of the failure slope component \( m_{gps}^2 \) described in Eq. (23);
- \( \bar{m}_{m-g}^2 \) denotes our SLAM-based IM estimate of the failure slope component \( m_{m-g}^2 \) described in the Eq. (23);
- \( \bar{m}^2 \) denotes our SLAM-based IM estimate of the worst-case total failure mode slope \( m^2 \), which is evaluated as the maximum of the individual worst-case failure slopes, namely, \( \bar{m}_{gps}^2 \) and \( \bar{m}_{m-g}^2 \). Note that, the component \( \bar{m}_{m-g}^2 \) is associated with a multiplying factor 2, which is based on our derivation in the Eq. (23).

Thereafter, we compute the first component of the failure mode slope, i.e., \( m_{gps}^2 \) as

\[
m_{gps}^2 = \frac{d_{gps}^T M_{gps}^T \zeta_{gps} \zeta_{gps}^T M_{gps} d_{gps}}{d_{gps}^T d_{gps}}
\]

with

\[
L_{gps} = \begin{bmatrix}
I^1_t & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & I^N_t \\
0_{8(N+1) \times 1} & \ldots & 0_{8(N+1) \times 1}
\end{bmatrix},
\]

where

- \( L_{gps} \) represents the fault matrix constructed from the fault probabilities calculated in the multiple FDI module, discussed earlier in the Section 2.6. Under the presence of multipath and considering \( N \) satellites in view, \( L_{gps} \) is given by Eq. (26);
- $M_{gps}$ denotes the residual matrix, such that, $M_{gps} = L_{gps}^T(I - K)^T(I - K)L_{gps}$;
- $d_{gps}$ denotes the transformed fault vector associated with the GPS pseudoranges, such that, $b_{gps} = L_{gps}M_{gps}d_{gps}$, where $b_{gps}$ denotes the fault vector and is defined earlier in Eq. (17);
- $\zeta_{gps}$ denotes the fault-probability-based gain matrix, such that, $\zeta_{gps} = L_{gps}^T K$, where $K$ is defined in Eq. (19).

The squared worst-case fault slope $\bar{m}_{gps}^2$ under fault hypothesis, computed in the Section 2.6, is equivalent to the maximum eigenvalue of the $N \times N$ matrix given by $M_{gps}^T \zeta_{gps} M_{gps}$. Therefore, the worst-case failure mode slope is given by

$$\bar{m}_{gps}^2 = \zeta_{gps}^T M_{gps}^T \zeta_{gps}. \quad (27)$$

Similarly, to compute the second component of the failure mode slope, i.e., $m_{m-g}$, we define the corresponding fault matrix, denoted by $L_{m-g}$ as

$$L_{m-g} = \begin{bmatrix} I_k^1 & I_8 & \cdots & 0_{8 \times 8N} & 0_{8 \times 8} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{8 \times 8N} & I_k^N & I_8 & 0_{8 \times 8} \\ 0_{16 \times 8} & \cdots & 0_{16 \times 8} & 0_{16 \times 8} \end{bmatrix}. \quad (28)$$

Thereafter, we rearranged the rows of the matrices $V$, $H$, $K$, $S^{-1}$, $R$, defined in Eq. (19), such that, their elements correspond to the elements of the fault matrix $L_{m-g}$, described in Eq. (28). Based on this, we compute the the worst-case failure slope of the second component $\bar{m}_{m-g}^2$ as

$$\bar{m}_{m-g}^2 = \zeta_{m-g}^T M_{m-g}^T M_{m-g} \zeta_{m-g}, \quad (29)$$

where $\zeta_{m-g} = L_{m-g}^T V R$ and $M_{m-g} = L_{m-g}^T K^T K L_{m-g}$.

Therefore, the worst-case total failure slope caused in the presence of multipath effects using our SLAM-based IM algorithm is given by

$$\bar{m}^2 = \max \left( \zeta_{gps}^T M_{gps}^T \zeta_{gps}, 2 \zeta_{m-g}^T M_{m-g}^T \zeta_{m-g} \right). \quad (30)$$

2) Worst-case failure slope under broadcast anomalies

Unlike multipath effects, during the occurrence of broadcast anomalies, the behavior of measurement error $r$ is dominated by the error in the satellite orbital model $\tilde{\theta}$ as compared to the measured pseudoranges $Z$. Therefore, $r_{gps} << r_{g-m}$ and $r_{mm} >> r_{m-g}$, based on which the failure mode slope $m^2$ described in Eq. (22) is written as

$$m^2 = \frac{\epsilon_{mm}^T r_{mm} + 2 \epsilon_{gps}^T r_{mm}}{r_{mm}^T r_{mm} + r_{g-m}^T r_{g-m}} + \frac{m_{m-m}^2 + 2 \kappa_2}{1 + \kappa_2}, \quad (31)$$

where

- $\kappa_2$ denotes the ratio between the measurement residuals, namely, $r_{g-m}$ and $r_{mm}$, and given by $\kappa_2 = \frac{r_{mm}^T r_{mm}}{r_{g-m}^T r_{g-m}}$;
- $m_{m-m}^2$ denotes the failure slope component obtained by considering the estimate error in the overall state vector caused by the motion model $\epsilon_{mm}$ and motion model component of the motion-model-based measurement residual $r_{mm}$, given by $m_{mm}^2 = \frac{\epsilon_{mm}^T \epsilon_{mm}}{r_{mm}^T r_{mm}}$;
- $m_{g-m}^2$ denotes the failure slope component obtained by considering the estimate error in the overall state vector, i.e., both $\epsilon_{gps}$ and $\epsilon_{mm}$ and motion-model component of the pseudorange residual, i.e., $r_{g-m}$. This is given by $m_{g-m}^2 = \frac{\epsilon_{gps}^T \epsilon_{mm}}{r_{g-m}^T r_{g-m}}$. 
Given the range of $\kappa_2$ is from $[0, \infty)$, during the presence of broadcast anomalies, our SLAM-based IM estimate of the worst-case total failure mode slope $m^2$, described in Eq. (22) is given by

$$\bar{m}^2 = \max(\bar{m}^2_{mm}, 2\bar{m}^2_{g-m})$$  \hspace{1cm} (32)

where

- $\bar{m}^2_{mm}$ denotes our SLAM-based IM estimate of the failure slope component $m^2_{mm}$ described in Eq. (23);
- $\bar{m}^2_{g-m}$ denotes our SLAM-based IM estimate of the failure slope component $m^2_{g-m}$ described in Eq. (23);
- $\bar{m}^2$ denotes our SLAM-based IM estimate of the worst-case total failure mode slope $m^2$, which is evaluated as the maximum of individual worst-case failure slopes, namely, $\bar{m}^2_{mm}$ and $\bar{m}^2_{g-m}$. Note that, the component $\bar{m}^2_{g-m}$ is associated with a multiplying factor 2, which is based on the derivation in Eq. (23).

We calculate the individual maximum failure mode slopes, i.e., $\bar{m}^2_{mm}$ and $\bar{m}^2_{g-m}$ in a similar manner as described in Section 2.7.1. To estimate $\bar{m}^2_{mm}$ and $\bar{m}^2_{g-m}$, we construct the corresponding fault matrices $L_{mm}$ and $L_{g-m}$, respectively, such that, $L_{mm} = L_{m-g}$ and $L_{g-m} = L_{gps}$ and thereafter, rearrange the matrices $V$, $H$, $K$, $S^{-1}$, $R$ accordingly. Based on this, the worst-case total failure slope caused by broadcast anomalies using our SLAM-based IM algorithm is given by

$$\bar{m}^2 = \max \left( \chi^T_{mm} M_{mm} M_{mm}^T \zeta_{mm}, 2\chi^T_{g-m} M_{g-m} M_{g-m}^T \zeta_{g-m} \right),$$  \hspace{1cm} (33)

where

- $M_{mm} = L_{mm}(I - VR)^T(I - VR)L_{mm}$;
- $\zeta_{mm} = L_{mm}^T VR$;
- $\zeta_{g-m} = L_{g-m}^T K$;
- $M_{g-m} = L_{g-m}^T (HVR)^T (HVR)L_{g-m}.$

3) Protection levels using worst-case failure slope

Based on the worst-case failure slope computed in the Eqs. (30) and (33), we calculate the protection level [41] for the given integrity requirements, i.e., probability of false alarm $P_{FA}$ and misdetection $P_{MD}$. Based on Fig. 1 in [41], for a given $P_{FA}$, under fault-free conditions, we calculate the threshold $\tau$ for the test statistic $q$ that follows a chi-squared distribution $\chi_N$ as

$$P_{FA} = p(q > \tau | q \sim \chi^2_N) = 1 - F_{\chi^2_N}(\tau).$$  \hspace{1cm} (34)

Thereafter, for a given $P_{MD}$, under fault conditions, we estimate the minimum detectable non-centrality parameter $\lambda_{det}$ of the non-centered chi-squared distribution that represents the test statistic $q$ as

$$P_{MD} = p(q \leq \tau | q \sim \chi^2_{(N, \lambda_{det})} = 1 - F_{\chi^2_{(N, \lambda_{det})}}(\tau).$$  \hspace{1cm} (35)

Finally, given a linear interpolation from the measurement residual to the position error, we calculate the protection level $PL$. The protection level $PL$ is equivalent to the $y$-coordinate that corresponds to the non-centrality parameter along the line passing through the origin and with slope given by $\bar{m}^2$.

$$PL = \lambda_{det} \sqrt{\bar{m}^2}$$  \hspace{1cm} (36)

Therefore, for a given pre-determined value of probability of false alarm and misdetection, we utilize our proposed SLAM-based IM algorithm, which not only detects and isolates multiple GPS faults, but also computes the protection levels associated with the estimated receiver position.
3 Experiments

We validate the proposed SLAM-based IM algorithm via three experimental scenarios. In the first set of experiments, by subjecting the GPS receiver mounted on a ground vehicle to simulated broadcast anomalies, we assess the fault probabilities associated with each satellite and thereby, detect and isolate multiple GPS faults. In the second set of experiments, using an aerial vehicle operating in urban environment, we demonstrate the capability of our algorithm to detect multipath-prone satellites as well as accurately estimate the receiver 3D position as compared to conventional least squares [12]. Finally, in the third set of experiments, using simulated receiver data, we validate the protection levels attained using our SLAM-based IM approach as compared to the traditional RCM [12].

![Figure 3: Data collection setup consisting of a GPS antenna and a USRP device, which is triggered by a CSAC [42].](image)

3.1 Implementation details

Our experimental setup is equipped with an AntCom 3GNSSA4-XT-1 GNSS antenna. We collected raw GPS samples using a Universal Software Radio Peripheral (USRP-N210) device, which is equipped with a DBSRX2 daughterboard and is connected to an external Microsemi Quantum SA.45s CSAC, as shown in Fig. 3.

During initialization, we estimate the empirical Gaussian error distribution of the incoming fault-free data or from other existing open-source non-faulty GPS data. In addition, we set the initial value of our satellite fault nodes to \( I_{t}^{1:N} = 0.5 \) indicating neutrality. Also, if the value of \( I_{t}^{i} < 0.55 \), we utilize its corresponding \( \gamma_{t}^{i} \) to update the distribution for the next time instant. We initialized the Huber M-estimator-based constant as \( k_{H} = 4.5 \), Bisquare constant as \( k_{B} = 3.9 \) and the damping factor in the LM algorithm as \( \beta_{0,0} = 1.2 \). For our experiments, we consider a constant velocity-based receiver motion model and determine the coefficients of the measurement noise covariance matrix \( Q \) by computing a least squares fit to the acceleration-time data of a generic vehicle [43].

3.2 Experiment 1: Multiple Satellite Broadcast Anomalies

Our first set of experiments are conducted on a moving ground vehicle under open-sky conditions, with 6 satellites in view. During the interval \( t = 15 \) s to \( t = 40 \) s, we added simulated broadcast anomalies to 3 satellites, namely, PRN 7, 11 and 28, thereby, inducing positioning errors of 3 km, 13 km and 6 km respectively. This is achieved by manipulating the orbital parameters obtained from the decoded navigation message. Given 4 satellites are required to estimate the navigation solution and there are 6 visible satellites, the conventional MHSS RAIM [28] described in the Section 1, fails to detect and isolate all the faulty satellites.

Utilizing our SLAM-based IM algorithm, we observed that the fault probability \( I_{t}^{i} \), seen in Fig. 4, increases from an Root Mean Squared Error (RMSE) of 0.28 to 0.57 for PRN 7, 0.36 to 0.81 for PRN 11 and 0.19 to 0.61 for PRN 28 during the presence of fault. In addition, our algorithm also accurately estimates that PRN 1 remains non-faulty at all times as it exhibits a low RMSE fault probability of 0.22 for the entire time duration. Therefore, we demonstrated that our algorithm not only detects and isolates multiple faults but also accurately estimates the low fault probability associated with the non-faulty satellites.
Figure 4: Fault probability $I_i$ associated with $i^{th}$ satellite is shown for 4 satellites, of which PRN 1 is non-faulty and the others i.e., PRN 7, 11 and 28 suffer from satellite broadcast anomalies during the interval $t = 15$ s to $t = 40$ s. Our SLAM-based IM algorithm accurately detects the presence of anomalies which is observed via an increase in the faulty probability above $I_i > 0.5$, as indicated by the red line.

Figure 5: ROC curve associated with the detection of fault in PRN 11. Using our SLAM-based IM algorithm, for a given probability of false alarm $P_F = 1e^{-6}$, the corresponding probability of detection $P_D = 0.77$.

In addition, for one satellite, namely, PRN 11, based on the empirical CDF described in Eq. (16), we plotted the Receiver Operating Characteristic (ROC) curve on a log10 scale, as seen in Fig. 5. We observed that for a given probability of false alarm $P_F = 1e^{-6}$, the corresponding successful probability of fault detection associated with PRN 11 using SLAM-based IM algorithm shows a high value of $P_D = 0.77$.

3.3 Experiment 2: Multipath Effects in an Urban Area

We conducted our next set of experiments in Champaign, Illinois by mounting a GPS antenna on the aerial vehicle shown in Fig. 6(a). To comply with the FAA regulations, we flew an aerial vehicle inside a netted cage mounted on the back of a flat-bed truck shown in Fig. 6(b). Our SLAM-based IM algorithm assesses the fault probability associated with each satellite and also adaptively utilizes this information to simultaneously localize both the receiver and satellites in view.
Figure 6: Experiment conducted in Champaign, Illinois using an aerial vehicle shown in (a), by flying it inside the netted cage mounted on the back of a flat-bed truck shown in (b). The experimental data captured the 3D dynamics of the aerial vehicle and GPS multipath caused by surrounding tall buildings.

In Fig. 7, we showed four snapshots at $t = 60$ s, $70$ s, $74$ s and $82$ s, which indicate the probability of fault in each satellite, computed using our SLAM-based IM algorithm. We plotted 4 out of 8 visible satellites, namely, PRN 8, 9, 22 and 26, with green indicating $I_i^t < 0.5$, i.e., low fault probability and red indicating $I_i^t > 0.5$, i.e., high probability. At $t = 60$ s, the receiver is not surrounded by buildings and therefore, all the 4 satellites exhibit low probability of fault as seen in Fig. 7(a). However, at $t = 74$ s, as seen in Fig. 7(c), 3 satellites namely, PRN 8, 9 and 22 are blocked by tall buildings and therefore, suffer from receiver faults. Similarly, at $t = 70$ s and $t = 82$ s, as seen in Fig. 7(b) and 7(d), our SLAM-based IM algorithm accurately captures the high fault probability in one and two satellites, respectively.

In Fig. 8, we observed that the conventional least squares, indicated in red, showed large total positioning errors with RMSE $11.18$ m and standard deviation of $33.56$ m. However, our SLAM-based IM algorithm, indicated in blue, demonstrated small receiver localization errors with RMSE $1.93$ m and standard deviation of $3.97$ m.

3.4 Experiment 3: Multipath effects in a simulated urban area

We conducted our final set of experiments by utilizing the simulated receiver data of Chicago, as seen in Fig. 9(a). We simulated the authentic raw GPS signals received at the ground vehicle using an online-available GPS simulator [44], at a sampling rate of $2.5$ MHz. During post-processing, from $t = 60$ s to $t = 120$ s, we induced multipath errors in the pseudoranges that correspond to the satellites with low-elevation and azimuth, i.e., in the range of $60 - 120^\circ$ and $240 - 300^\circ$ with respect to the geographic North, namely, PRNs 12, 18, 20 and 25, as seen in Fig. 9(b). To induce multipath effects, we introduced uncorrelated biases in pseudoranges by considering single-point reflections from the continuous planar surface of the urban buildings with random pre-determined heights.
Figure 8: Receiver localization errors estimated using our SLAM-based IM are indicated in blue and least squares are shown in red. Scalar tracking showed large total positioning errors with RMSE 11.18 m and standard deviation of 33.56 m whereas our SLAM-based IM, demonstrated low receiver localization errors with RMSE 1.93 m and standard deviation of 3.97 m.

Figure 9: Details about the experiment setup. (a) trajectory followed by the receiver in downtown Chicago where the blue highlighted area represents the presence of dense multipath; (b) skyplot indicating the GPS satellites observed by the receiver, PRNs 12, 18, 20 and 25 are induced with multipath effects when the receiver in the highlighted blue area.

In Fig. 10, for the total time of $t = 0 - 120$ s, the positioning error computed using the conventional least squares with a total RMSE of 19.03 m and standard deviation of 13.6 m, is indicated by the red-solid line and the SLAM-based IM algorithm with a total RMSE of 6.59 m and standard deviation of 5.86 m is depicted by the blue-solid line.

The horizontal protection levels associated with the estimated receiver position using the traditional maximum failure slope described in [28] is indicated by the red-dashed line and using our SLAM-based IM algorithm, as described in the Section 2.7, is indicated by the blue-dashed line. Therefore, during multipath, i.e., $t = 60 - 120$ s, we observed that tighter protection levels are attained using our SLAM-based IM algorithm, i.e., with an RMSE of 17.82 m and standard deviation 4.33 m as compared to the traditional approach, i.e., with RMSE of 70.24 m and standard deviation 21.95 m.
CONCLUSIONS

We proposed a Simultaneous Localization and Mapping (SLAM)-based Integrity Monitoring (IM) algorithm that not only evaluates the protection levels associated with the estimated receiver position, but also detects and isolates multiple GPS faults caused by multipath and satellite broadcast anomalies. Inspired by SLAM for robotics, we estimate the PVT of GPS satellites in a 3D environment while simultaneously localizing the GPS receiver within it. In particular, we designed a Graph-SLAM framework, where we utilized the sequential GPS pseudoranges, receiver and satellite motion model to optimize the graph using an M-estimator-based Levenberg Marquardt algorithm. Later, we assessed the probability of fault associated with each satellite by evaluating the corresponding test statistic against an on-the-fly empirical estimation of its CDF. We also theoretically compute the protection levels of the receiver position by evaluating the worst-case failure slope that maximizes the eigenvalues of the dominant GPS faults.

Under simulated multipath effects, we validated that tighter horizontal position protection levels are achieved using our SLAM-based IM algorithm, which showed an RMSE of 17.82 m as compared to the traditional RAIM approach, which showed an RMSE of 70.24 m. We also validated the performance of our SLAM-based IM by adding simulated broadcast anomalies to the collected open-sky GPS data. We have shown that our algorithm accurately estimates a low fault probability, with an RMSE of 0.22, for a non-faulty satellite and high fault probability of 0.57, 0.81 and 0.61 for the 3 faulty satellites. In the presence of multipath, we demonstrated that our SLAM-based IM adaptively estimates the multipath-prone satellites and accurately localizes the receiver with RMSE 1.93 m and standard deviation of 3.97 m.

REFERENCES


